都市空間における交通混雑の分析と次善の料金政策

（課題番号：13630008）

平成13・14・15年度
科学研究費補助金 基盤研究（C）⑵
研究成果報告書

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目　次

第１章　はじめに

都市における交通混雑を軽減するため，日本に限らず多くの国の大都市では，これまで数十年にわたって交通施設の拡張や新設などが行われてきたが，混雑の緩和は遅々として進んでいない．交通混雑は代表的な外部不経済であり，それを内部化する政策なしに道路容量のみを増やしてもその効果は小さく，場合によっては社会全体の経済厚生を低下させる場合すらある．この問題に対する経済学からの提言は，混雑の外部不経済を内部化する混雑料金の導入である。しかし混雑の外部効果は，都市空間における各地点ごとに異なるので、実際に料金制を導入するには「どこで」「いくらの」料金を課すべきかを決めねばならない。

1960年代から現在にいたるまで、交通経済学において交通混雑に関する膨大な数の研究があるが、その多くは一点経済を対象とした非空間モデルに基づいている。そもそも交通現象が、離れた地点間の空間的移動であることを考えれば、空間を考慮しない交通問題の分析は非現実的である。一方,都市経済学では、1970年代以降、混雑が存在する状況での土地利用の空間均衡モデルが開発され、最善(First-best)の混雑料金が導出されている(たとえばSolow and Vickrey(1971), Anas(1996))。このような最善の料金は、立地点ごとに異なった額となり、それを実際に徴収することは技術的にも社会的合意の点からも不可能といえる。そこで混雑料金を実施できない状況で、道路容量を最適に調節するという次善の政策に関する研究が行われている（たとえばKanemoto(1980)）。これまでの研究では、厳密に最善の混雑料金か、全く実施しないかという両極端のケースの分析に終始している。また従来のモデルでは、土地利用に対する効果にのみ注目しており、交通需要が非弾力的であると仮定されている点も制約的である。

本研究では、都市の空間構造を考慮して、交通混雑の理論モデルを構築するとともに、それを用いて代替的な混雑対策の効果を経済学的に分析する。具体的には、次善の料金政策であるコードンプライシングに着目する。これらは、厳密には最適な料金システムではないが、実施が容易である。実際、コードンプライシングはシンガポールやノルウェーの3都市ですでに実施されている。これらの事例において料金水準やコードンの位置は、経済分析にもとづいて最適に設定されているわけではない。本研究では混雑の外部性による経済損失を最小化するような料金の額やコードンの位置を決定する条件式を導出し、その性質について理論的分析を行う。さらには実際の都市に

おける交通流動のデータを用いて、料金システムの効果を定量的に評価することを目的とする。

本報告書の構成は次の通りである。

第２章では、単一中心都市におけるコードンプライシングに関する分析を行う。コードンプライシングとは、混雑の激しい都市中心部を取り囲むようにコードンラインを設け、そのコードンを通過する車両に対して定額の料金を課するものである。単一中心都市においては、すべての交通が都心に向かうものと仮定される。ここではコードンの位置と料金水準の最適な組み合わせに関する条件を導出し、その性質について理論的に分析する。

第３章では、前章の分析を非単一中心都市を対象とするものに拡張する。非単一中心都市では、各地点から都市内のすべての地点へのトリップの可能性を考慮する。

第４章では、より現実的な分析を行うため、大阪都市圏における道路ネットワークを対象として、コードンプライシングの導入効果を分析する。ここでは実際のデータに基づいてトリップ需要関数、費用関数のパラメータを推定し，シミュレーションを通じて最適なコードン料金額を求め、そのもとでの経済厚生を評価する。また多重コードンを想定し料金の最適な組み合わせについても検討している。

第 2 章　単一中心都市における最適なコードンプライシング

Optimal Cordon Pricing in a Monocentric City*

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Abstract:

This paper presents a simple spatial model of traffic congestion for a monocentric city to investigate the effects of cordon pricing on trip-making and congestion level in each location. Optimal cordon pricing is obtained as a combination of the cordon location (i.e. distance of the cordon from the CBD) and the amount of toll charged there that maximizes the total social surplus in a city. Under optimal cordon pricing, trips originating from locations inside the cordon are under-priced, those just outside the cordon are over-priced and those near the urban fringe are under-priced. Numerical simulations using the parameter values based on Japanese data suggest that cordon pricing attains an economic welfare level very close to the first-best optimum.

Running head: Cordon Pricing

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1. INTRODUCTION

Cordon pricing is adopted in most practices of road pricing to control area-wide congestion in a city (e.g. Singapore, Hong Kong, Oslo, etc.)[1]. A typical cordon pricing system is designed as follows: each vehicle is charged a fixed toll when it passes through the specified cordon surrounding the central area of a city where traffic is most congested. Although this scheme is not the first best pricing rule for congestion management, the system is simple and easy to implement. However, it is not an easy task for a traffic control authority to rationally determine the toll level and the cordon location, since it should take into account the distortions in the market that are commonly present in the second best world. For this reason, it is unclear whether particular toll levels in the actual cases are too high or low, or whether the sizes of cordoned areas are too large or small.

This paper presents a formal economic analysis dealing with the following issues of cordon pricing: where the cordon line should be located; at what level the toll should be set. Recently, an increasing number of researchers have approached this problem using network models. May et al. [8] computed the effects of alternative road pricing schemes including cordon pricing by applying a network simulation model to the city of Cambridge, U. K. They merely examined the consequences of exogenously specified cordon locations and toll levels. Santos, et al. [10] used the same network simulation model to obtain the optimal cordon tolls for eight English towns. They did not discuss the optimal locations of cordons. The above studies were chiefly concerned with empirical estimates of toll levels and social welfare, so theoretical analysis on qualitative properties of traffic pattern and resource allocation under cordon pricing has not been presented. On the other hand, Verhoef [15], Zhang and Yang [16] discussed the mathematical problem to obtain the optimal choices of toll levels and locations of toll collection in a network. They mainly focused on methods to compute the optimal solutions, and presented the numerical results for hypothetical example networks[2]. Although network models are useful for practical applications, they are not suitable for investigating the general properties of the problem, since the results depend on the network structures specified for calculations. We need a model that explicitly deals with the spatial patterns of trip making behavior and traffic congestion in an idealized setting, such as the continuous space model developed in urban economics literature. Such an approach may provide common insights into the design of cordon pricing.

Urban economists have developed urban spatial models incorporating congestion effects and

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[1] Small and Gomez-Ibañez [11] provide an overview of various practices of road pricing across the world.

[2] Recent paper by de Palma and Lindsey [2] considers the problem of finding the optimal number and locations of a tolled road in the radial network. Although they provide some interesting policy implications, the analysis is based on artificial setting in that all residents are located on a single circumference (same distance from the center).

discussed the discrepancy between equilibrium and optimal land use (e.g., Anas and Xu [1], Fujita [3], Kanemoto [4], Sullivan [12]). They showed that a congestion toll (or location tax) internalizing congestion externalities should be charged to each resident to achieve the first-best optimal allocation. Congestion externalities vary depending on locations: the levels of tolls should be differentiated by residential location. Obviously, the implementation of such a tolling policy is practically infeasible, so second best policies should be considered instead. In this direction, Kanemoto [4] focused on the problem of how road capacity at each location should be determined in the absence of a toll. Most of the earlier works in urban economics, however, consider two extreme pricing schemes, rigorous first best toll and no toll. In other words, the second-best pricing policies in the context of urban space have not been sufficiently explored[3]. Sullivan [13] and Kraus [5] are exceptions. Sullivan examined a second best policy in which the toll is proportional to the distance traveled (this is effectively the same as fuel tax), based on the general equilibrium simulation model of urban land use. Kraus numerically calculated the welfare gains from various pricing regimes including cordon pricing. To our knowledge, Kraus's paper is the only work that examines the effect of cordon pricing based on the urban spatial model. In the present paper, unlike Kraus, we explicitly solve the spatial patterns of trip generation, and this enables us to describe the situations of resource allocations at different locations in a city.

Another limitation in the literature on urban economics is that trip demand is assumed to be inelastic. In this case, tolling policies have no effect in the short-run where land use is unchanged. Instead, we focus on the roles of tolling policies as instruments to control trip demand generation and its spatial distribution. So, our model allows elastic trip demand while land use is assumed fixed. Each resident chooses how often to travel; the choice is affected by trip cost that varies with location of traveller.

This paper discusses the optimal combination of cordon location and toll level in a monocentric city. We investigate how cordon pricing affects trip demand and resource allocation at each location in a city, and evaluate the performance of the optimal cordon pricing in terms of social welfare by comparison with the welfare levels under no-toll equilibrium, or the first-best optimum toll. We further examine the effects of parameter changes by numerical simulations.

2. THE MODEL

2-1 Trip demand and cost

Suppose a linear city where CBD is located at the center (origin of coordinate) and residential areas

[3] On the other hand, there exists extensive literature on second best pricing in non-spatial setting (e.g., Liu and McDonald [6], Marchand [7], Verhoef, et al. [14]).

are developed around the CBD, the land area of the CBD is negligible and the characteristic of each residential location is represented solely by distance from the CBD. It is assumed that all residents in the city are homogenous and population density is unity throughout the urban area. Each individual makes trips for some purpose from his/her residence to the CBD by car; other types of trip are neglected. Trip demand is elastic; depending on the cost of car trips, each individual chooses the number of car trips within a certain period (sometimes he/she gives up a trip or uses an alternative mode of transportation such as public transit). Let x be the distance from the CBD, and $q(x)$ the trip demand of a resident located at x, then the (inverse) trip demand function is given as,

$$p(q(x)) = a - bq(x), \tag{1}$$

where a, b are positive constants. $p(q(x))$ represents the private marginal benefit of a trip.

We assume that the cost for driving the unit distance around x is an increasing function of the traffic volume there, $Q(x)$, which is denoted by $t(Q(x))$. This implies that the road width is constant at all locations. Then, the cost for a trip from x, $C(x)$, is given as,

$$C(x) = \int_0^x t(Q(y))dy. \tag{2}$$

Since all trips are destined for the CBD and population density is unity, traffic volume at x, $Q(x)$ is defined as,

$$Q(x) = \int_x^B q(y)dy, \tag{3}$$

where B is the distance from the CBD to the edge of the urban area.

In this paper, the functional form of $t(Q(y))$ is specified as follows:

$$t(Q(y)) = f + cQ(y), \tag{4}$$

where f and c represent, respectively, the cost for driving the unit distance when the traffic volume is zero (= at free speed), and the marginal cost with respect to traffic volume.

2-2 No-toll equilibrium

Each individual decides to make a trip as long as the private marginal benefit exceeds the private cost. In equilibrium, the following relation holds at all locations,

$$p(q(x)) = C(x) \qquad \text{for all } x, \quad 0 \le x \le B, \tag{5}$$

together with Eqs. (2) and (3).

By substituting Eqs. (1) and (2) into Eq. (5), and differentiating both sides of the obtained equation with respect to x, we have,

$$-bq'(x) - t(Q(x)) = 0, \tag{6}$$

where $q'(x)$ is the first derivative of $q(x)$. Next, differentiating both sides of Eq. (3) with respect to

x yields the following,

$$Q'(x) = -q(x). \tag{7}$$

Differentiating Eq. (6) once more and incorporating the relations of Eqs. (4) and (7), we obtain the following differential equation,

$$-bq''(x) + cq(x) = 0, \tag{8}$$

where $q''(x)$ is the second derivative of $q(x)$. It turns out that $q'(x) < 0$ from (6), and $q''(x) > 0$ from (8)[4]. In other words, the number of trips per person at each location decreases with the distance from the CBD, and the rate of decrease diminishes.

The equilibrium number of trips generated at each location, $q^*(x)$, is obtained by solving the differential equation (8), as follows:

$$q^*(x) = \lambda_1 \exp(\alpha x) + \lambda_2 \exp(-\alpha x), \tag{9}$$

where $\alpha = \sqrt{c/b}$, and λ_1, λ_2 are unknown constants to be determined by boundary conditions.

We need two boundary conditions: the first is $q^*(0) = a/b$, which is derived by evaluating Eq. (5) at $x = 0$ and applying $C(0) = 0$; the second is $-bq^{*\prime}(0) = f + cQ(0)$ from Eq. (6) at $x = 0$. With these two relations, λ_1, λ_2 are determined as follows:

$$\lambda_1 = \frac{a\exp(-\alpha B) - f/\alpha}{b(\exp(\alpha B) + \exp(-\alpha B))},$$

$$\lambda_2 = \frac{a\exp(\alpha B) + f/\alpha}{b(\exp(\alpha B) + \exp(-\alpha B))}.$$

2-3 First-best optimum

The first-best optimum is defined as the trip pattern that maximizes total social surplus in a city, formulated as follows:

$$S = \int_0^B \left[\int_0^{q(x)} p(q)dq - C(x)q(x) \right] dx. \tag{10}$$

From the optimal conditions, we have the following relation,

$$p(q(x)) = C(x) + \int_0^x t'(Q(y))Q(y)dy, \tag{11}$$

where the second term of the RHS represents the congestion externalities that an additional trip from

[4] As shown later, differential equations of trip rate functions for first-best optimum and equilibrium under cordon pricing have the same structure as Eq. (8). Thus, $q'(x) < 0$ and $q''(x) > 0$ also hold for these schemes.

x imposes on all drivers using the road between x and 0. Therefore Eq. (11) is consistent with the general rule for social efficiency: The social marginal benefit from an additional trip at location x should be equalized to the social marginal cost. Such socially efficient allocation can be decentralized by levying the toll equal to the congestion externality on each trip generated at each location. Implementing this tolling scheme is not practically feasible since toll levels should be differentiated at each location. This case does not provide an alternative policy but serves as a reference point for evaluating the performance of cordon pricing as a second-best policy.

Using the specifications of Eqs. (1) and (4), Eq. (11) is rewritten as,

$$a - bq(x) = fx + 2c\int_0^x Q(y)dy \, . \tag{12}$$

As in the no-toll equilibrium, differentiating twice the above equation with respect to x yields,

$$-bq''(x) + 2cq(x) = 0 \tag{13}$$

and following the same procedure as above, optimal trips at each location, $q^o(x)$, are obtained as,

$$q^o(x) = \eta_1 \exp(\gamma x) + \eta_2 \exp(-\gamma x), \tag{14}$$

where $\gamma = \sqrt{2c/b}$, $\eta_1 = \dfrac{a\exp(-\gamma B) - f/\gamma}{b(\exp(\gamma B) + \exp(-\gamma B))}$, $\eta_2 = \dfrac{a\exp(\gamma B) + f/\gamma}{b(\exp(\gamma B) + \exp(-\gamma B))}$.

3. EQUILIBRIUM UNDER CORDON PRICING

Suppose that the cordon is located at distance x_m from the CBD, and a toll equal to τ is levied on each vehicle passing the cordon. In this situation, trip cost for a resident living outside the cordon $(x > x_m)$ is the sum of the travel time cost and toll, while a resident inside the cordon incurs only travel time cost. Let $q_i^{**}(x)$ and $q_o^{**}(x)$ represent the equilibrium number of trips departing at x inside and outside the cordon, respectively. Equilibrium requires that the following relations hold,

$$\begin{cases} p\left(q_i^{**}(x)\right) = C(x) \\ C(x) = \int_0^x t\left(Q_i(y)\right)dy \\ Q_i(y) = \int_y^{x_m} q_i^{**}(z)dz + \int_{x_m}^B q_o^{**}(z)dz \end{cases} \qquad for \quad 0 \le x \le x_m \tag{15}$$

$$\begin{cases} p\left(q_o^{**}(x)\right) = C(x) + \tau \\ C(x) = \int_0^{x_m} t\left(Q_i(y)\right)dy + \int_{x_m}^{x} t\left(Q_o(y)\right)dy \\ Q_o(y) = \int_y^B q_o^{**}(z)dz \end{cases} \quad for \quad x_m \le x \le B \tag{16}$$

$$Q_i(x_m) = Q_o(x_m). \tag{17}$$

Eqs. (15) and (16) give two differential equations describing the spatial variation of trips, but these are connected to each other by Eq. (17), which states that traffic volume function should be continuous at x_m.

As in the previous section, we use specifications $t(Q) = f + cQ$ and $p(q) = a - bq$, to solve the differential equations. Then, the equilibrium trip rate functions under cordon pricing are derived as,

$$q_i^{**}(x) = \mu_1 e^{\alpha x} + \mu_2 e^{-\alpha x}, \quad \text{for } 0 \le x \le x_m, \tag{18a}$$

$$q_o^{**}(x) = \mu_3 e^{\alpha x} + \mu_4 e^{-\alpha x}, \quad \text{for } x_m \le x \le B, \tag{18b}$$

where α has been defined in the previous section, and $\mu_1, \mu_2, \mu_3, \mu_4$ are unknown constants to be determined by boundary conditions as follows.

We need four boundary conditions. At $x = 0$, the following relations hold,

$$p\left(q_i^{**}(0)\right) = C(0) = 0, \tag{19a}$$

$$p' \frac{dq_i^{**}(0)}{dx} - t\left(Q_i(0)\right) = 0. \tag{19b}$$

The next condition is derived by incorporating Eq. (17) into Eqs. (15) and (16) at $x = x_m$; that is,

$$p(q_i^{**}(x_m)) = p(q_o^{**}(x_m)) - \tau. \tag{19c}$$

Finally, at the edge of the urban area, $x = B$,

$$p' \frac{dq_o^{**}(B)}{dx} - t\left(Q_o(B)\right) = 0. \tag{19d}$$

From the above conditions (19a)-(19d), unknown constants are determined as follows:

$$\mu_1 = \frac{-2\frac{f}{\alpha} + 2ae^{-\alpha B} + \tau\left(e^{\alpha(B-x_m)} - e^{-\alpha(B-x_m)}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)}, \tag{20a}$$

$$\mu_2 = \frac{2\frac{f}{\alpha} + 2ae^{\alpha B} - \tau\left(e^{\alpha(B-x_m)} - e^{-\alpha(B-x_m)}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)}, \tag{20b}$$

$$\mu_3 = \frac{-2\frac{f}{\alpha} + 2ae^{-\alpha B} - \tau\left(e^{-\alpha(B-x_m)} + e^{-\alpha(B+x_m)}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)}, \tag{20c}$$

$$\mu_4 = \frac{2\dfrac{f}{\alpha} + 2ae^{\alpha B} - \tau\left(e^{\alpha(B+x_m)} + e^{\alpha(B-x_m)}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)}. \tag{20d}$$

The effects of exogenous changes in cordon location and toll level on the equilibrium number of trips in each location are given as follows:

$$\frac{\partial q_i^{**}(x)}{\partial x_m} = \frac{-\tau\left(\alpha e^{\alpha(B-x_m)} + \alpha e^{-\alpha(B-x_m)}\right)\left(e^{\alpha x} - e^{-\alpha x}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)} < 0, \quad \text{for } 0 \le x \le x_m, \tag{21a}$$

$$\frac{\partial q_o^{**}(x)}{\partial x_m} = \frac{-\tau\alpha e^{-\alpha(x+x_m)}\left(-e^{2\alpha B} + e^{2\alpha(B+x_m)} - e^{2\alpha x} + e^{2\alpha(x+x_m)}\right)}{2b\left(1 + e^{2\alpha B}\right)} < 0, \quad \text{for } x_m \le x \le B, \tag{21b}$$

$$\frac{\partial q_i^{**}(x)}{\partial \tau} = \frac{\left(e^{\alpha x} + e^{-\alpha x}\right)\left(e^{\alpha(B-x_m)} - e^{-\alpha(B-x_m)}\right)}{2b\left(e^{-\alpha B} + e^{\alpha B}\right)} > 0, \quad \text{for } 0 \le x \le x_m, \tag{22a}$$

$$\frac{\partial q_o^{**}(x)}{\partial \tau} = -\frac{e^{-\alpha(x+x_m)}\left(e^{2\alpha B} + e^{2\alpha x}\right)\left(1 + e^{2\alpha x_m}\right)}{2b\left(1 + e^{2\alpha B}\right)} < 0, \quad \text{for } x_m \le x \le B. \tag{22b}$$

(21a) and (21b) state that, as the cordon location moves outward, the equilibrium number of trips decreases in all locations[5]. (22a) and (22b) state that, as the toll level rises, the number of trips increases in locations inside the cordon but decreases outside the cordon. This also implies that in inner (outer) locations, the numbers of trips under cordon pricing are larger (smaller) than those under no-toll equilibrium. This is illustrated in Fig. 1. Trip demand decreases in outer locations due to additional toll burden, causing a reduction in traffic volume (in other words, congestion level) in inner locations. Trip makers in inner locations enjoy congestion relief with no charge, then respond by increasing trips. In other words, the cordon pricing induces increase in consumer surplus for residents in inner locations and decrease in consumer surplus for those in outer locations.

(21a) implies that outward move of the cordon location causes aggravation of traffic congestion in inner locations, while trip demand decreases in all locations. To see this, we examine the change in traffic volume at location x, ($0 \le x \le x_m$) caused by an infinitesimal change in the cordon location from x_m to $x_m + dx_m$.

$$\frac{\partial Q_i(x)}{\partial x_m} = \int_x^{x_m} \frac{\partial q_i^{**}(y)}{\partial x_m} dy + \int_{x_m}^{B} \frac{\partial q_o^{**}(y)}{\partial x_m} dy + q_i^{**}(x_m) - q_o^{**}(x_m) \tag{23}$$

Although the first and second terms of the RHS are negative from (21), the third term is larger than the fourth term. Trip makers located between x_m and $x_m + dx_m$ increase trips from $q_o^{**}(x_m)$ to $q_i^{**}(x_m)$ because they are exempt from the toll after the move. This increase in trip demand exceeds

[5] Rigorously speaking, there is an exception: as shown later, the equilibrium number of trips at x_m increases because it switches from $q_o(x_m)$ to $q_i(x_m)$, and $q_o(x_m) < q_i(x_m)$ as shown by (22).

the sum of the trip decrease over all locations[6]. When the cordon location moves outward, those who are exempt from the toll due to this change are better off, while the others are worse off.

Figure 1

4. THE OPTIMAL CORDON PRICING

Optimal cordon pricing is the combination of the cordon location x_m and toll τ that maximizes the social surplus defined as follows:

$$S = \int_0^{x_m}\left[\int_0^{q_i^{**}(x)} p(q)dq - C(x)q_i^{**}(x)\right]dx + \int_{x_m}^{B}\left[\int_0^{q_o^{**}(x)} p(q)dq - C(x)q_o^{**}(x)\right]dx .$$

Constraints to this problem are Eqs. (15)-(17), i.e., the equilibrium conditions under cordon pricing. Note that the equilibrium number of trips under cordon pricing is solved explicitly as $q^{**}(x)$ in the last section. Hence, we can treat this problem as optimization without constraints by substituting (18) and (19) into the objective function above.

The optimal conditions with respect to x_m and τ are given respectively as follows:

$$\int_0^{q_i^{**}(x_m)} p(q)dq - C(x_m)q_i^{**}(x_m) - \left[\int_0^{q_o^{**}(x_m)} p(q)dq - C(x_m)q_o^{**}(x_m)\right]$$
$$+\int_0^{x_m}\left[p(q_i^{**}(x)) - C(x) - E(x)\right]\frac{\partial q_i^{**}(x)}{\partial x_m}dx + \int_{x_m}^{B}\left[p(q_o^{**}(x)) - C(x) - E(x)\right]\frac{\partial q_o^{**}(x)}{\partial x_m}dx = 0 \tag{24a}$$

$$\int_0^{x_m}\left[p(q_i^{**}(x)) - C(x) - E(x)\right]\frac{\partial q_i^{**}(x)}{\partial \tau}dx + \int_{x_m}^{B}\left[p(q_o^{**}(x)) - C(x) - E(x)\right]\frac{\partial q_o^{**}(x)}{\partial \tau}dx = 0, \tag{24b}$$

where

$$E(x) = \begin{cases} \int_0^{x} t'(Q_i(y))Q_i(y)dy, & \text{for } 0 < x \le x_m \\ \int_0^{x_m} t'(Q_i(y))Q_i(y)dy + \int_{x_m}^{x} t'(Q_o(y))Q_o(y)dy & \text{for } x_m < x \le B \end{cases}. \tag{25}$$

$E(x)$ is the sum of the congestion externality that an additional trip from x imposes on all drivers

[6] This is verified by expanding the RHS of Eq. (23), as follows:

$$\frac{\partial Q_i(x)}{\partial x_m} = \frac{\alpha\left(-e^{\alpha(2B-x-x_m)} + e^{\alpha(2B+x-x_m)} - e^{-\alpha(x-x_m)} + e^{\alpha(x+x_m)}\right)}{2\left(1+e^{2\alpha B}\right)} > 0$$

using the road between x and 0.

The first line of Eq. (24a) represents the direct effect on social surplus caused by outward move of the cordon location x_m: Increase in consumers' surplus for those who are exempt from the toll minus decrease in toll revenue[7]. It is easily seen that the sum of the terms on the first line has positive value. Integral terms on the second line of (24a) represent the sum of indirect effects on social surplus through changes in the number of trips caused by outward move of the cordon location. In other words, these effects are considered as changes in the amount of dead weight losses present in the second-best situation. The first integral describes the effects in locations inside the cordon, which has positive value in view of Eqs. (15) and (21a)[8]. This positive effect on social surplus implies that the sum of the dead weight losses inside the cordon decreases as the cordon moves outward. Based on the discussion so far, the second integral term should be negative for Eq. (24a) to hold. Recalling $\frac{\partial q_o^{**}(x)}{\partial x_m} < 0$ from Eq. (21b), $[p(q_o^{**}(x)) - C(x) - E(x)]$ in the second integral should have positive values for, at least, some locations between x_m and B.

Likewise, the first integral term on the LHS of Eq. (24b) has negative value from (22a) and (15): Dead weight losses increase in locations inside the cordon as the toll rises. For Eq. (24b) to hold, the second integral should be positive. Since $\frac{\partial q_o^{**}(x)}{\partial \tau} < 0$ from (22b), $[p(q_o^{**}(x)) - C(x) - E(x)]$ in the second integral must have negative values for, at least, some locations between x_m and B.

Synthesizing the above discussions, it turns out that, in locations outside the cordon, $p(q_o^{**}(x)) - C(x) - E(x)$ has negative values in some locations and positive values in other locations. Note that $p(q_o^{**}(x)) - C(x) - E(x) = \tau - E(x)$ from Eq. (16), and $E(x)$ is monotonously increasing with x. Thus, only the configuration as illustrated in Figure 2 is possible: There exists some point $\tilde{x}$

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[7] Note that outward move of cordon location reduces the private cost for those located at x_m from $C(x_m)+\tau$ to $C(x_m)$. The first line of (24a) can be rewritten as follows:

$$\left[\int_0^{q_i^{**}(x_m)} p(q)dq - C(x_m)q_i^{**}(x_m)\right] - \left[\int_0^{q_o^{**}(x_m)} p(q)dq - \{C(x_m)+\tau\}q_o^{**}(x_m)\right] - \tau q_o^{**}(x_m).$$

Two bracketed terms are consumer surplus for trip makers located just inside and outside of the cordon, respectively. Thus, they represent increase in consumer surplus for those located at x_m. The third term is the amount of toll charged to a trip maker located just outside the cordon, which is foregone revenue due to move of the cordon location.

[8] Since $p(q_i^{**}(x)) - C(x) = 0$ from (15), the bracketed term in the first integral on the second line of (24a) becomes $-E(x)$, which is negative. And $\frac{\partial q_i^{**}(x)}{\partial x_m} < 0$ from (21a). Therefore, the first integral has positive value.

such that $\tau - E(x)$ is positive between x_m and $\tilde{x}$ but negative outside $\tilde{x}$.

Figure 2

Based on the above discussion, the situations of resource allocation under cordon pricing for three typical locations are illustrated in Fig. 3. $C(x)+E(x)$ represents the social marginal cost, i.e., increment of social cost due to marginal increase in trips from x. In locations inside the cordon $(x < x_m)$, the social marginal cost exceeds the social marginal benefit, $p(q_i^{**}(x))$, in other words, trips are under-priced, or the number of trips is larger than the efficient level. Similarly, trips are over-priced for $x_m < x < \tilde{x}$, and under-priced for $\tilde{x} < x < B$. The amount of dead weight losses due to inefficient trip making as discussed above is equal to the shaded areas in the figure. As the cordon location or toll level is changed, dead weight loss is increased in some locations and decreased in other locations.

Figure 3

Table 1 summarizes the directions of changes in dead weight losses due to changes in the cordon location and toll at their optimal levels. In the table, sign + (-) means that dead weight loss is increased (decreased) by increasing the corresponding control variable (i.e., x_m or τ) from their optimal levels.[9] For example, at locations within $0 \le x < x_m$, dead weight loss is decreased as x_m increases (the cordon location moves outward). Optimal cordon pricing is designed so that the gains from the decrease in dead weight loss offset the losses from the increase in dead weight loss.

Table 1

5. EVALUATION OF ECONOMIC WELFARE

This section numerically examines the effects of cordon pricing on economic welfare, by comparing the values of social surplus under optimal cordon pricing, no-toll equilibrium and first-best optimum. To obtain quantitative insights, we calibrated parameter values of the model using the actual data for

[9] Note that increase in dead weight loss means decrease in social surplus.

Osaka Prefecture, Japan, as follows[10]:

B=50, a = 130, b =498, c = 0.52, f =1.2.

Table 2 summarizes the result for these parameter values (shown as basic case). Optimal cordon location is 7.54 km from the CBD, and the time-equivalent value of the optimal toll is 29.42 minutes. The amount of the toll is 980.7 Yen, if we adopt 2000 Yen/hour for the value of travel time, which is the estimate from an empirical study in Japan (Ohta [9]). The table also shows values of social surplus under three schemes, which are measured in time equivalent units[11]. From the table, social surplus for cordon pricing is larger than that for no-toll equilibrium by 12%, and smaller than the first-best optimum by only 0.7%. Although the cordon pricing is a very simple system, the performance is almost as good as the first-best optimum that requires prohibitive effort for information processing.

Table 2

Does the above result depend on the specific parameter values used here? We carried out simulations for various parameter values to check the robustness of the results against the parameter changes. Table 2 shows the results for different values of parameters in demand and cost functions. Larger b implies that trip demand is less elastic. Smaller c means that the congestion level is less sensitive to an increase in traffic volume, which is interpreted as a situation that road capacities are larger at all locations. The table shows that, as trip demand is less elastic, the optimal cordon location becomes farther from the center and the toll level is lower. On the other hand, smaller c causes a larger-sized cordoned area and lower toll level. Figures 4 and 5 plot the social surpluses under no-toll equilibrium, optimal cordon pricing and first-best optimum for various values of b and c.

Figure 4

Figure 5

Figures show that the values of social surpluses for optimal cordon pricing are very close to those for the first-best optimum regardless of parameter values. This suggests that cordon pricing attains an economic welfare level nearly as good as the first-best optimum for a wide range of parameter values.

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[10] Sources of data and details of procedure to calibrate parameter values are given in the Appendix.

[11] Note that parameter values are specified in the context of the model setting: City is monocentric, population density is unity, etc. Therefore, the values of social surpluses are meaningful only for comparison among different tolling schemes.

Why does the cordon pricing produce such a good result as shown above? Let us investigate in more detail the workings of cordon pricing as a device to control congestion externality. Note that, as seen in Eq. (25), congestion externality depends on traffic volume at each location that is integral of trips originating in outer locations. Fig. 6 plots traffic volumes $Q(x)$ for no-toll equilibrium, first-best optimum and cordon pricing. By definition (Eq. (3)), traffic volume at the edge of the urban area, B, is equal to zero in all cases, and the (negative) slope of each curve is equal to the trip rate originating at each location, $q(x)$.

Figure 6

The figure shows that the traffic volume curve for the cordon pricing closely matches the curve for the first-best optimum, and two curves cross twice at intermediate locations. Recall that, under the optimal cordon pricing, trips are under-priced in locations inside the cordon, over-priced just outside the cordon and under-priced in the fringe of the urban area. Accordingly, the trip rate under cordon pricing tends to be larger (smaller) than that for the first-best optimum in locations where congestion is under-priced (over-priced). This is reflected in the relative steepness of the two curves in the figure; traffic volume curve for the cordon pricing is steeper inside the cordon, flatter just outside the cordon and steeper in the fringe of the urban area. Although cordon pricing is such a simple system in which tolls are collected at only one point, it divides the urban area into three zones and fine-tunes the trip rate in each zone (=slope of traffic volume curve) to minimize the deviation of traffic volume from the first-best.

6. CONCLUSION

This paper presents a simple spatial model of traffic congestion for a monocentric city to investigate the effects of cordon pricing on trip-making and congestion level in each location. Optimal cordon pricing is obtained as a combination of the cordon location (i.e. distance of the cordon from the CBD) and the amount of toll charged there that maximizes the total social surplus in a city. Under the optimal cordon pricing, trips from locations inside the cordon are under-priced, those just outside the cordon are over-priced and those in the fringe of the urban area are under-priced. Numerical simulations using the parameter values based on Japanese data suggest that the cordon pricing attains an economic welfare level very close to the first-best optimum.

This paper introduces a number of assumptions to simplify the analysis. The most restrictive one is that the city is monocentric: All trips are destined to the CBD. If this assumption is relaxed, the result that cordon pricing attains good performance may be modified significantly. In this case, it might be necessary to introduce multiple cordons. We should also consider the land use change to see the long-run effects. Since trips departing at locations inside the cordon are exempt from tolls, central locations become more attractive under cordon pricing. This induces land use structure with higher density in central locations, which is likely to have positive impacts on efficiency. As shown in the literature of monocentric city models with congestion, efficiency is improved when the physical size of the city becomes more compact (e. g., Fujita [3], Kanemoto [4]). This implies that the centralization of land use caused by cordon pricing improves the efficiency of spatial distribution in a city. When we relax the assumption of monocenter, however, different forms of land use changes may occur. Firms and households may move to locations outside the cordon, thereby completing trips without crossing the cordon, i.e., avoiding tolls. In other words, introducing cordon pricing induces the dispersion of spatial structure unlike centralization in the monocentric case. And such changes may weaken the effectiveness of the policy. This problem is worth investigating in future works. Anas and Xu [1] provide a useful prototype framework for the analysis of this problem, although they do not consider the second-best pricing policies. We have also neglected the costs of setting up the cordon pricing system[12]. It is natural that the set-up costs tend to increase as the cordon moves outward, since the number of road links crossing the cordon increases. Therefore, optimal location of the cordon becomes closer to the CBD than that obtained in this paper. Other important issues to be addressed by future study include investment for road capacity; as Kanemoto [4] discussed, naive benefit-cost criterion is not applicable to the second-best situations. Since all these extensions will make the model structure too complicated, we need to rely on numerical methods.

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APPENDIX: Parameters for numerical analysis

We used the data from "Person Trip Survey for the Keihanshin Area" in 1990, which include information of origin, destination and travel time for each individual trip on a given day. The data are aggregated by 67 jurisdictions in Osaka prefecture, and then the number of trips from each jurisdiction to the CBD (defined as Kita-ku and Chu-o-ku of Osaka City) is extracted. Aggregated numbers of trips are divided by populations of jurisdictions to obtain the number of trips per person. Distances from the centers of jurisdictions to the CBD are measured on the map.

Parameters to be calibrated are B, a, b, c, and f.

First, we assume that the distance from the CBD to the edge of the urban area, B, is 50 km, considering that the southern edge of the Osaka area (Misaki Cho) is located 59 km from the CBD while the northern edge (Nose Cho) is 39 km away.

Parameter a in the demand function is interpreted as the travel time at which one gives up making trips. We set a = 130, in reference to the fact that the longest travel time among trips to the CBD reported in the trip survey is equal to 120 minutes. Since the demand function is linear, b is obtained by drawing a line connecting two points: One is the intercept (0,a) on the number of trips - travel time plane, the other one is the observation at the zone where the number of trips per person is largest in the study area. Then, we have $b = 498$. Parameter setting in the above manner implies that the trip cost is measured in time units (minutes). Thus, the amount of toll is also computed in time units. The amount of toll in monetary units is obtained by multiplying the value of time.

Parameter f in the cost function represents the time to drive the unit distance in free-flow traffic, i.e., when traffic volume is equal to zero. Assuming that speed in free-flow is equal to 50 km/h, it follows that f = 1.2. Parameter c is determined so that the trip pattern computed by the model best fits the actual pattern, although the model is built on a number of assumptions that are not necessarily compatible with the situation of an actual city, such as monocentricity, unit density, constant road capacity, etc. By substituting already determined parameter values, B, a, b and f, to Eq. (9), we have $q^*(x)$ as a function of x and c. And substituting this into Eq. (3), $Q(x)$ is obtained as a function of x and c. Let x_i be the distance from zone i to the CBD, then in the context of the model, the travel time from zone i to the CBD, $T(x_i,c)$, is computed by the following formula,

$$T(x_i,c) = fx_i + c\int_0^{x_i} Q(y)dy .$$

Finally, c is determined so that the sum of square errors is $\sum_{i=1}^{67}\left[T(x_i,c)-\overline{T}_i\right]^2$, where $\overline{T}_i$ is the observed travel time from i. It follows that $c = 0.52$.

Table 1 Changes in dead weight losses under optimal cordon pricing

	locations		
	$0 \leq x < x_m$	$x_m < x < \tilde{x}$	$\tilde{x} < x \leq B$
x_m	-	+	-
τ	+	+	-

Table 2 Numerical results

	Basic case	Larger b	Smaller c
b in demand function	498	748	498
c in cost function	0.52	0.52	0.26
Optimal cordon location	7.54 km	8.32 km	8.78 km
Optimal toll	29.42 min	25.63 min	22.65 min
Social surplus:			
No-toll equilibrium;	233.5 min (0.887)	184.6 min (0.918)	309.0 min (0.939)
First-best optimum;	263.4 min (1.000)	201.0 min (1.000)	329.2 min (1.000)
Cordon pricing;	261.6 min (0.993)	200.0 min (0.995)	327.9 min (0.996)

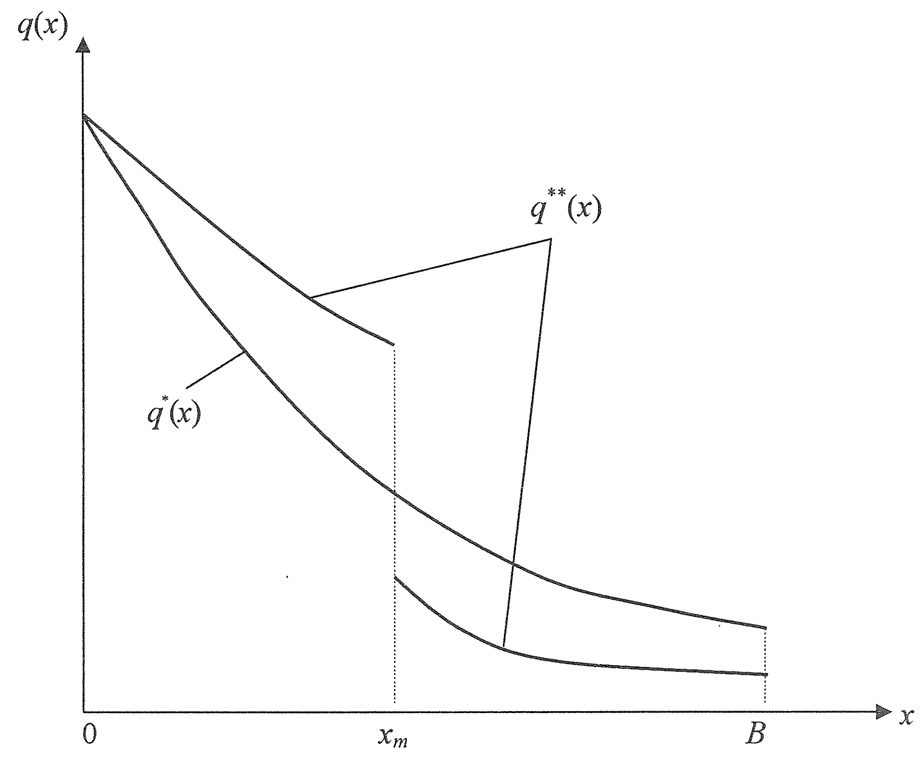


Figure 1 Spatial variations of trip rates under no-toll equilibrium and cordon pricing

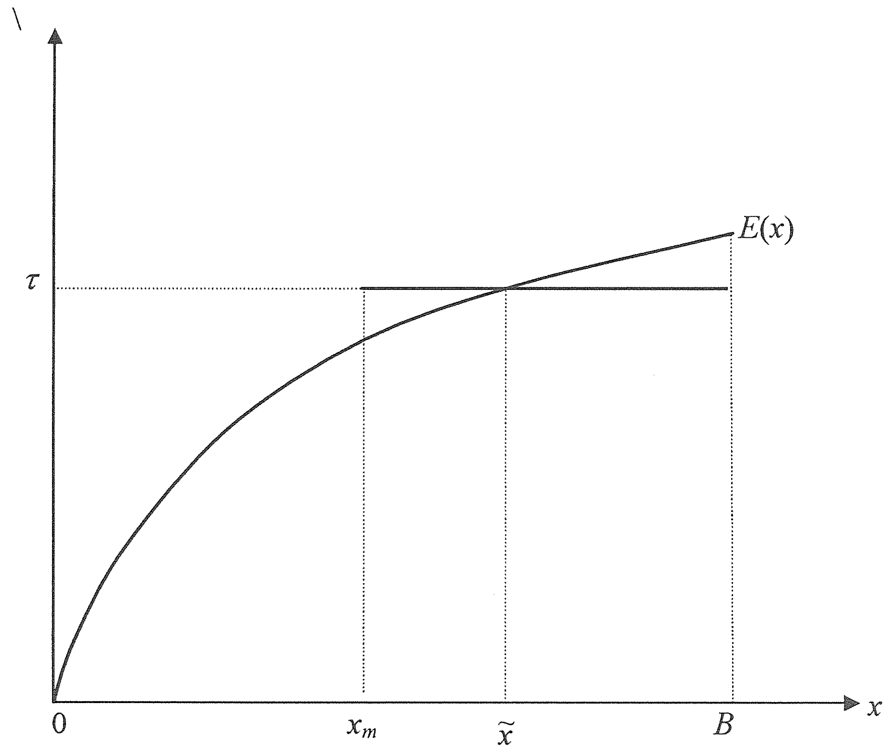
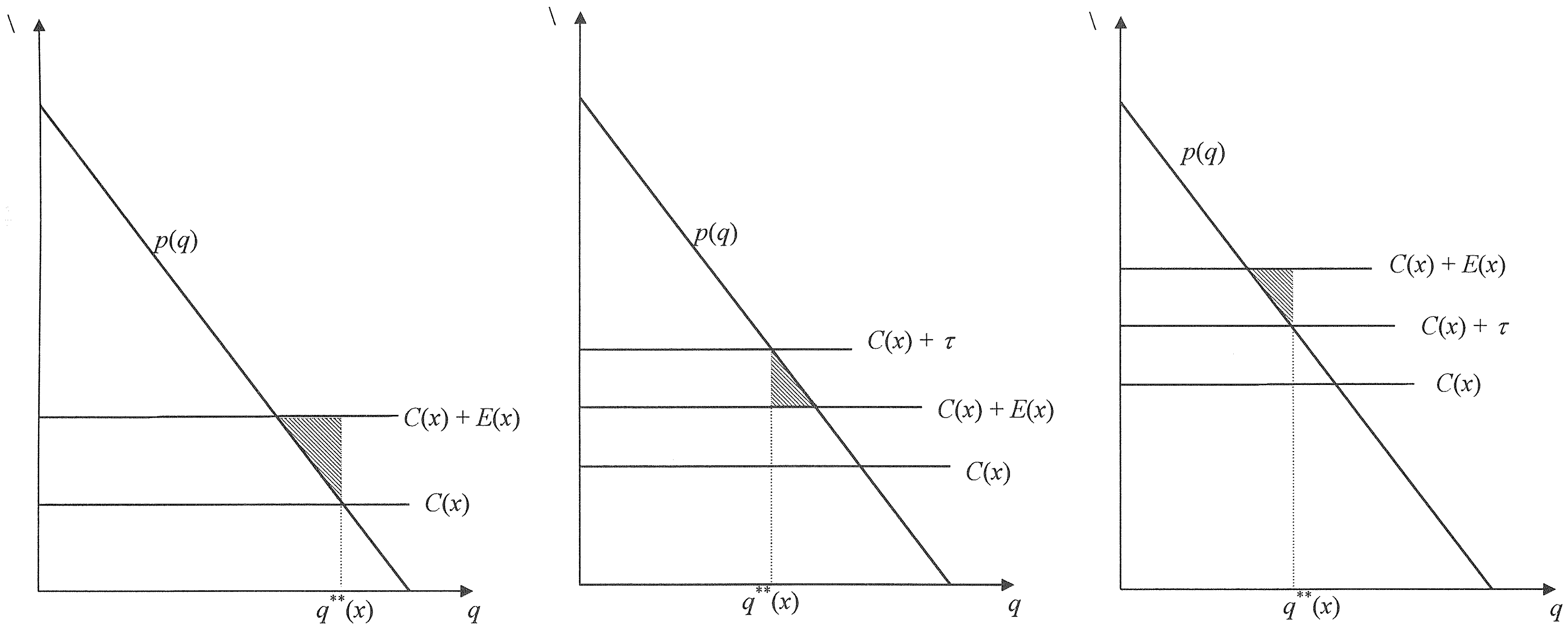


Figure 2 External cost and toll under the optimal cordon pricing



(a) $0 \le x \le x_m$ (b) $x_m \le x \le \tilde{x}$ (c) $\tilde{x} \le x \le B$

Figure 3 Relations among trip costs, tolls and external costs in three representative locations

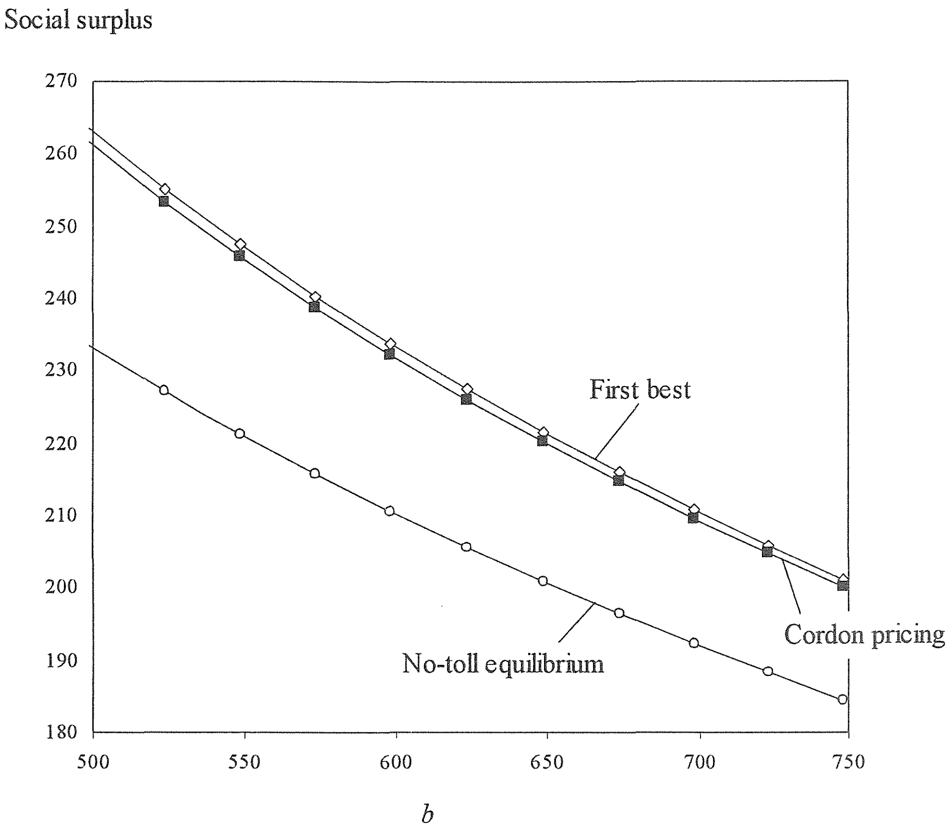


Figure 4 Demand elasticity parameter b and economic welfare

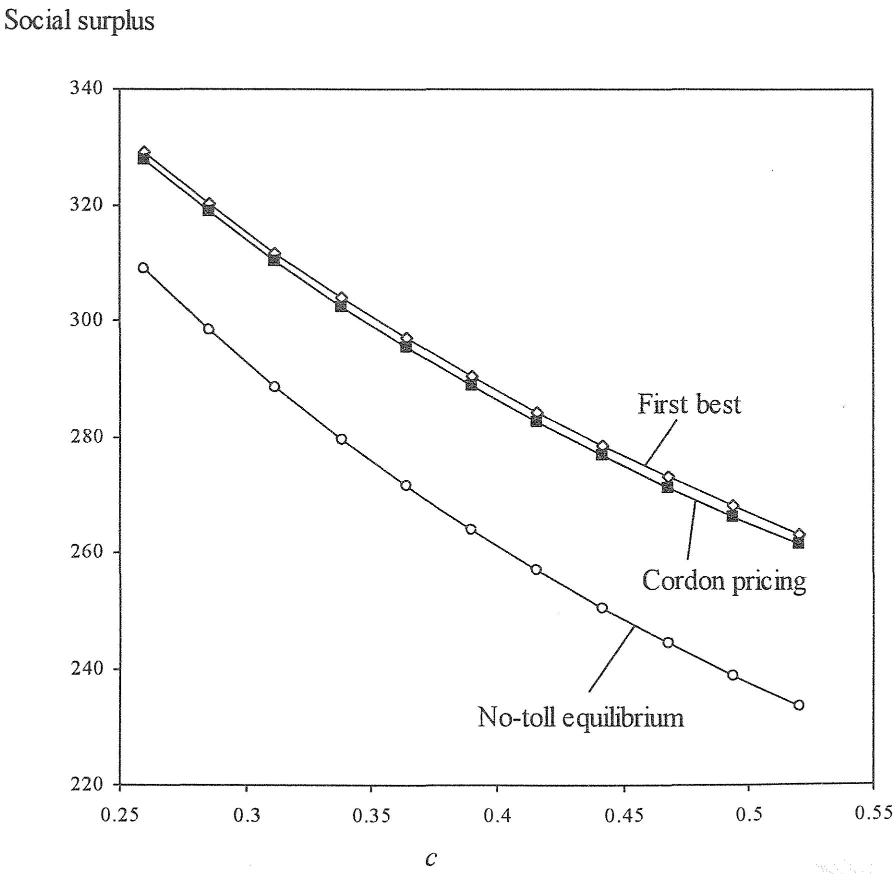


Figure 5 Cost parameter c and economic welfare

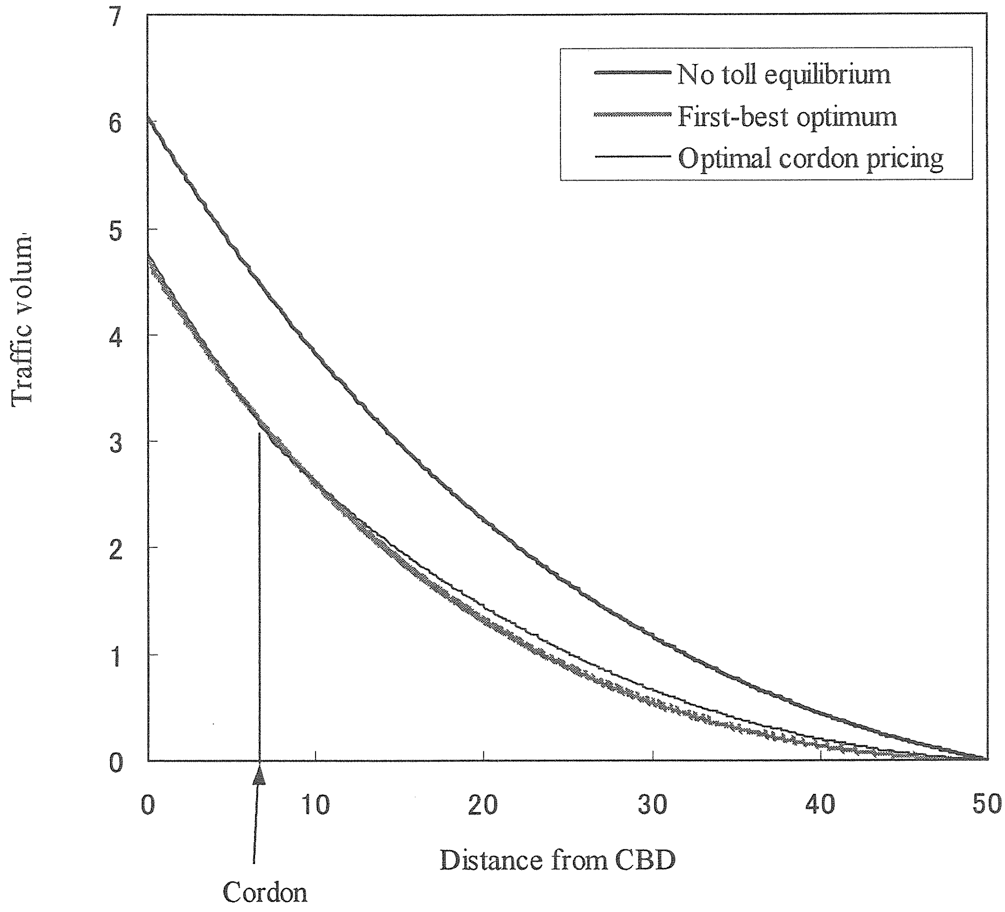


Figure 6 Profiles of traffic volume

第3章　非単一中心都市における最適なコードンプライシング

Optimal Cordon Pricing in a Non-Monocentric City*

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Abstract:

This paper examines the effect of cordon pricing based on urban spatial model of non-monocentric city where trips may occur between any pairs of locations in a city. The model describes spatial distribution of trip demand and traffic congestion under alternative pricing schemes. We evaluate the efficiency of resource allocation by comparing three schemes: no-toll equilibrium, first-best optimum, and optimal cordon pricing. Optimal cordon pricing is defined as combination of cordon location and toll level that maximizes the social surplus in a city. Simulations show that cordon pricing is not always effective for congestion management: the cordon pricing tends to be effective as urban structure is closer to mono-centric.

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1. Introduction

In recent years there has been increasing interest in cordon pricing as a measure to control traffic congestion in an urban area. It was reported that implementations in some cities such as Singapore and three Norwegian cities (Oslo, Bergen, Tronheim) were successful[1]. Policy makers in cities suffering heavy traffic congestion now consider the cordon pricing as a promising policy alternative. This situation induced many research works aiming to evaluate the effects of cordon pricing, or develop methods to obtain the optimal design of the pricing system (e.g., May and Milne (2000), Santos, Newbery, Rojey (2000), Verhoef (2002), Zhang and Yang (2002)). These works are mainly based on network models, with which results depend on network structure specified for simulations. Furthermore, they did not discuss the effect of land use structure of cities. Santos, Newbery, Rojey (2000) calculate the optimal cordon tolls for eight English towns, and report that the effects of cordon pricing are considerably different among eight towns. These differences in the effectiveness should be attributed to differences in network structures and land use patterns among cities. It is worth examining the effects of these factors on the effectiveness of pricing policies in idealized setting, such as continuous space models in urban economics literature (e.g., Kanemoto (1980), Sullivan (1983), Fujita (1989), Kraus (1989), Anas and Xu (1999)).

Mun, Konishi and Yoshikawa (2003) investigate the effect of cordon pricing based on urban spatial model of monocentric city, and show that the cordon pricing attains an economic welfare level very close to the first-best optimum[2]. The system works as follows: under the optimal cordon pricing, the urban area is divided into three zones with respect to the distance from the center, and trips from locations inside the cordon are under-priced, those just outside the cordon are over-priced and those in the fringe of the urban area are under-priced. Cordon location and toll fine-tune the trip rate in each zone to minimize the deviation of traffic volume from the first-best. This model is based on a number of assumptions to obtain analytical solution: linear demand, uniform density, uniform road capacity, etc. It is unclear how relaxing these assumptions affect the results.

This paper extends the analysis to deal with the situation in a non-monocentric city. Unlike monocentric city where all trips are destined to CBD, trips may occur between any pairs of locations in a non-monocentric city. We develop a model to describe spatial distribution of trip

[1] London started congestion charge in the central area in 2003. The system is "Area pricing", which is similar but different to cordon pricing.

[2] Ho, Wong, Yang, Loo (2003) obtained the similar result in the setting of two-dimensional continuum traffic network.

demand and traffic congestion under alternative pricing schemes. The model also allows variable density of land use and road capacity across space. We evaluate the economic welfare under three schemes: no-toll equilibrium, first-best optimum, and optimal cordon pricing. Optimal cordon pricing is defined as combination of cordon location and toll level that maximizes the social surplus in a city. We examine the effects of urban spatial structures and various parameters on traffic patterns and effectiveness of cordon pricing. Such information may provide useful insights for policy makers.

2. The model

2-1 No-toll equilibrium

We assume a city developed on a one-dimensional space such as the long-narrow city in Solow and Vickrey (1971), in which only trips in the lengthwide direction are concerned. Each location is represented by coordinate value on the one-dimensional space, with the center specified as origin (see Figure 1). Homogenous individuals are located according to a given density function such that the density is highest at the center. Each individual makes trips to various locations. Trip demand is elastic; frequency of trips depends on trip cost that consists of time cost and toll. Time cost is affected by congestion levels along the course of trip.

Figure 1

Let $q(x,y)$ denote the number of trips an individual in location x makes to location y. No-toll equilibrium of trip distribution is characterized as the situation that private marginal benefit equal trip time cost for every O-D pair.

$$P(q(x,y)) = C(x,y) \qquad \text{for all } x, y \tag{1}$$

where $P(q)$ is inverse demand function that represents private marginal benefit of trips, and $C(x,y)$ is time cost for a trip from x to y. $C(x,y)$ is formulated as follows

$$C(x,y) = \int_x^y t\left(\frac{Q(z)}{L(z)}\right)dz \tag{2}$$

where $t(Q(z)/L(z))$ is time required to drive unit distance around location z, $Q(z)$ and $L(z)$ are respectively traffic volume and road capacity at z. The function $t(\bullet)$ is increasing with respect to traffic volume - capacity ratio. Traffic volume is the sum of trips passing location z along the route of trips.

$$Q(z)=\int_{-B}^{z}\int_{z}^{B}n(x)q(x,y)dydx+\int_{z}^{B}\int_{-B}^{z}n(x)q(x,y)dydx \tag{3}$$

where $n(x)$ is population at x, B and $-B$ are the right and left fringes of the city measured by distance from the center. The first term of RHS represents the sum of trips passing z from left to right, while the second term represents that from right to left.

2-2 First-best optimum

The first-best optimum is attained when the marginal social benefit equals to social marginal cost for each O-D pair, as follows

$$P\left(q(x,y)\right)=C(x,y)+E(x,y), \quad \text{for all } x,y \tag{4}$$

$$E(x,y)=\int_{x}^{y}t'\left(\frac{Q(z)}{L(z)}\right)\frac{Q(z)}{L(z)}dz. \tag{5}$$

where $E(x,y)$ represents the congestion externality that an additional trip from x imposes on all drivers using the road between x and y. Although the first best solution is achieved by levying tolls equal to congestion externalities, implementing such tolling scheme is not practically feasible since toll levels should be differentiated by O-D pairs. This case does not provide an alternative policy but serves as a reference point for evaluating the performance of cordon pricing as a second-best policy.

3. Optimal cordon pricing in a non-monocentric city

Suppose that the cordon is located at distance x_m from the center, and a toll equal to τ is levied on each vehicle passing the cordon. Equilibrium under cordon pricing is characterized by the following condition

$$P(q(x,y))=C(x,y)+J(x,y)\tau, \qquad \text{for all } x, y \tag{6}$$

where $J(x,y)$ is number of times crossing the cordon along the route of trip from x to y, which is defined as follows

$$J(x,y)=\left|h(x)-h(y)\right| \tag{7}$$

where $h(x)=-1$ for $-B\le x<-x_m$

$h(x)=0$ for $-x_m\le x<x_m$

$h(x)=1$ for $x_m\le x<B$

For some O-D pairs, such as trips from the left hand side of $-x_m$ to the right hand side of x_m, drivers have to pay the toll two times[3].

The optimal cordon pricing is obtained by solving the problem to maximize total social surplus with respect to toll level and cordon location,

$$\underset{x_m,\,\tau}{Max}\ \int_{-B}^{B} n(x)\int_{-B}^{B}\left[\int_{0}^{q(x,y)} P(q)\,dq - C(x,y)q(x,y)\right]dydx \tag{8}$$

subject to equilibrium conditions (6) (7).

Considering that there are only two control variables, we obtain the optimal solution by grid search.

4. Simulations

4-1 Specifications of functional forms and parameters

Demand function is specified as follows

$$q(x,y) = am(y)\exp\left(-bP(x,y)\right) \tag{9}$$

where a, b are positive constants, $m(y)$ is intensity of economic activities (e.g., population, employments) at location y that represents attractiveness of destination. $P(x,y)$ is the full price (generalized cost) of a trip from x to y that consists of time cost and toll. Aggregated number of trips from x to y is obtained as $an(x)m(y)\exp\left(-bP(x,y)\right)$, which is equivalent to exponential type gravity model. By inverting the demand function (9) we have the marginal benefit of trips, as follows[4]

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[3] Since we assume a one-dimensional space, it is impossible to choose the routes avoiding the toll payment, such as detour around the cordon line.

[4] The (inverse) demand function (10) is derived from the following utility function,

$$U = X + \frac{a}{b}\int_{-B}^{B}\int_{0}^{m(y)}\left[s(x,y,j) - \left(\frac{s(x,y,j)}{a}\right)\ln\left(\frac{s(x,y,j)}{a}\right)\right]djdy \tag{i}$$

where X is consumption of composite good that does not involve trips, $s(x,y,j)$ is the number of trips that an individual located in x makes to j-th destination located in y. The above utility function implies that individual enjoys higher utility as he/she visit various destinations. Utility maximization subject to the budget condition, $I = X + \int_{-B}^{B}\int_{0}^{m(y)} P(x,y)s(x,y,j)djdy$, yields the following relation,

$$P(x,y) = -\frac{1}{b}\ln\left(\frac{s(x,y,j)}{a}\right) \tag{ii}$$

Since the specification (i) implies that the destinations in y are equally attractive, we have $s(x,y,j) = q(x,y)/m(y)$. Inserting this relation into (ii), we have (10).

$$P(q(x,y)) = -\frac{1}{b}\ln\left(\frac{q(x,y)}{am(y)}\right) \tag{10}$$

We estimate the parameter values using the trip survey data in Osaka prefecture, Japan. Then we have $a = 1.48 * 10^{-7}$, $b = 0.0312$.

Population density function $n(x)$ is specified as negative exponential form, widely applied in the literature (e. g., McDonald (1989)).

$$n(x) = d\exp(-gx) \tag{11}$$

where $d = 24500$, $g = 0.0854$.

Road capacity $L(x)$ is also specified as exponential function,

$$L(x) = k\exp(-vx) \tag{12}$$

We used the data of (road area / land area) ratio as a proxy of road capacity, and obtained the estimates of parameters as $k = 0.24$, $v = 0.0446$.

Travel time for unit distance is assumed to be linear with respect to volume-capacity ratio,

$$t\left(\frac{Q(x)}{L(x)}\right) = f + c\left(\frac{Q(x)}{L(x)}\right)$$

where $f = 1.2$, $c = 0.00000552$. We estimated these values applying the similar method to that in Mun, Konishi, Yoshikawa (2003). f is interpreted as time required to drive 1 km under zero flow condition; $f = 1.2$ implies that free-flow speed is 50 km/h.

Finally, the distance from the center to city fringe, *B*, is assumed to equal 50 km.

4-2 Results for base case

We conduct numerical simulations to evaluate the relative performances of three schemes. For the base case, it is assumed that $m(y) = n(y)$. The result for the base case is summarized as follows

$x_m = 13$ km $\qquad \tau = 19$ min

$SS_{Notoll} = 7.731\text{E+}08$ min $\qquad SS_{Cordon} = 8.008\text{E+}08$ min $\qquad SS_{Firstbest} = 8.397\text{E+}08$ min

$$\text{Gain from cordon pricing} = \frac{\mathrm{SS_{Cordon}} - \mathrm{SS_{No\ toll}}}{\mathrm{SS_{No\ toll}}} = 0.036$$

$$\text{Maximal gain} = \frac{\mathrm{SS_{First\ best}} - \mathrm{SS_{No\ toll}}}{\mathrm{SS_{No\ toll}}} = 0.086$$

$$\text{Relative gain} = \frac{SS_{\text{Cordon}} - SS_{\text{No toll}}}{SS_{\text{First best}} - SS_{\text{No toll}}} = 0.415$$

where SS_{Notoll}, SS_{Cordon}, and $SS_{Firstbest}$ are the time equivalent values of social surpluses for no-toll equilibrium, optimal cordon pricing, and first-best optimum, respectively. The optimal cordon location is 13km from the center and the toll in time equivalent unit is 19 minutes. The amount of the toll is 633.3 Yen (or $5.81), if we adopt 2000 Yen/hour for the value of travel time, which is the estimate from an empirical study in Japan. The effectiveness of cordon pricing is evaluated by two indexes: one is the "Gain from cordon pricing" representing improvement from no-toll equilibrium, another is "Relative gain" representing the ratio of gain from cordon pricing to maximal gain achievable in the first-best optimum. In the base case, cordon pricing improves the social surplus by 3.6% compared with the no-toll equilibrium, which accounts for 41.5% of maximal gain. Relative gain of the cordon pricing is much lower than that reported in Mun, Konishi and Yoshikawa (2003) that assumes monocentric city, uniform desnsity, linear demand, etc.

Figure 2 shows the spatial variation of congestion levels that are represented by traffic volume-capacity ratio. The figure shows that cordon pricing lowers the congestion levels around the cordon location ($x = 13$ and $x = -13$). In other locations congestion levels under cordon pricing are not very different from those under no-toll equilibrium. This suggests that trip-makers substitute among the destinations of trips. To see this, we take a closer look at changes in O-D trips.

Figure 2

Figure 3 describes deviations of trips under cordon pricing from the no-toll equilibrium and the first-best optimum for representative O-D pairs. In the figure, signs indicated in the parentheses, $(d1, d2)$, describe the deviations of trips as follows

$$d1 = sgn\left(q^{**}(x,y) - q^{O}(x,y)\right)$$
$$d2 = sgn\left(q^{**}(x,y) - q^{*}(x,y)\right)$$

where $q^{*}(x,y)$, $q^{O}(x,y)$, $q^{**}(x,y)$ are O-D trips under no-toll equilibrium, first-best optimum, optimal cordon pricing, respectively. Positive $d1$ means that trips are under-priced for the O-D pairs, and positive $d2$ means that number of trips under cordon pricing is larger than those under no-toll case. The figure shows that short trips crossing the cordon are over-priced while other trips are under-priced. This property is consistent with that obtained in monocentric case (Mun, Konishi, Yoshikawa (2003)). It is also observed that, by introduction of cordon pricing, all O-D trips crossing the cordon are reduced, while those without crossing the cordon are increased. This implies that trip makers switch the destinations to avoid paying the toll. As the sum of these individual responses, traffic volume around the cordon are reduced, even smaller than the first-best optimum.

Figure 3

Below we investigate the effects of urban structure and various parameters to find out when the cordon pricing does better job and when it does not.

4-3 Effects of urban spatial structure

In our model, spatial distributions of trip generation and destination are represented by the density functions $n(x)$ and $m(x)$ respectively. We examine the effects of urban spatial structure via differences in the gradient of the density function. The gradient of density function is represented by parameter g in (11). Suppose that the gradient of $m(x)$ becomes steeper holding the gradient of $n(x)$ unchanged. This change can be interpreted that urban spatial structure becomes closer to monocentric (see Figure 4). At the extreme, the city is completely monocentric when all trips are destined to CBD, as in Mun, Konishi, Yoshikawa (2003). This is expressed by the situation that g is extremely large.

Figure 4

We compute the solutions for various values of g in $m(x)$ holding the parameters in $n(x)$ fixed at the level in base case. When g in $m(x)$ is changed, another parameter value, d, is adjusted so that the value of $\int_{-B}^{B} m(x)dx$ is unchanged. This is to avoid scale effect and focus on pure effects of spatial distribution. Table 1 and Figure 5 shows the results of simulations.

Table 1

Figure 5

It is observed that the cordon pricing does better job as destinations of trips are more concentrated around the center. This suggests that the cordon pricing is effective in such cities where urban structure is close to mono-centric. While people can adjust their choices of trip destinations so as to avoid crossing cordon in a non-monocentric city, such responses are infeasible in monocentric city where all trips are destined to the center.

Next we look at overall gradient of population density. Gradients of $n(x)$ and $m(x)$ are changed in parallel. Both of gain from cordon pricing and relative gain are increasing with density gradient. Optimal cordon location is closer to the center as density gradient is higher. This is to capture more vehicles crossing the cordon.

Table 2

4-4 Effects of parameters

Table 3 shows the effects of demand elasticity. Demand elasticity is represented by parameter a in Eq. (10). As a is increased, the demand curve is rotated in anticlockwise direction. Thus larger value of a implies that trip demand is more elastic. It is observed from the table that both maximal gain and gain from cordon pricing increase with a. In other words, pricing policies are more effective as demand is more elastic. This is consistent with intuition, and the result obtained by Santos, Newbery, Rojey (2000). However, increments in the values of gain from cordon pricing are very small. As a consequence, index of relative gain is decreasing with a; cordon pricing is relatively ineffective in the case of elastic demand.

Table 3

Table 4 shows the effects of road capacity. We examine the uniform expansion of road capacity across the city area, which is represented by increasing the value of parameter k in Eq. (12).[5] Gains from pricing policies, represented by maximal gain and gain from cordon pricing, are smaller as road capacity is larger. Values of gain from cordon pricing are not very different among cases. On the other hand, relative gain of cordon pricing is increasing with road capacity.

Table 4

5. Conclusion

[5] Note that changes of k have the same effect as changes of c in Eq. (13).

This paper presents a spatial model of traffic congestion in a non-monocentric city, where trips occur between any pair of locations in a city. We evaluate the economic welfare for three schemes; no-toll equilibrium, first-best optimum, and optimal cordon pricing. Numerical simulations demonstrated that effectiveness of the cordon pricing depends on various factors including spatial structure of the city and other parameters. The results of the simulations are summarized as follows.

Welfare improvement by introducing the cordon pricing tends to be large, when

- the urban spatial structure is close to monocentric;
- density gradient is steeper;
- trip demand is less elastic;
- road capacity is larger.

Cordon pricing is not always effective. The above results suggest that the cordon pricing is effective in small size cities. According to the urban economics literature, spatial structure is likely to be monocentric, and density gradient is steeper, as city size is smaller. Furthermore, trip elasticity tends to be higher as availability of alternative travel modes is higher, of which large cities have the advantage. We have not sufficient empirical evidences on the relation between city size and road capacity. Policy makers should consider the suitability of the environments in choosing whether to adopt cordon pricing for congestion management.

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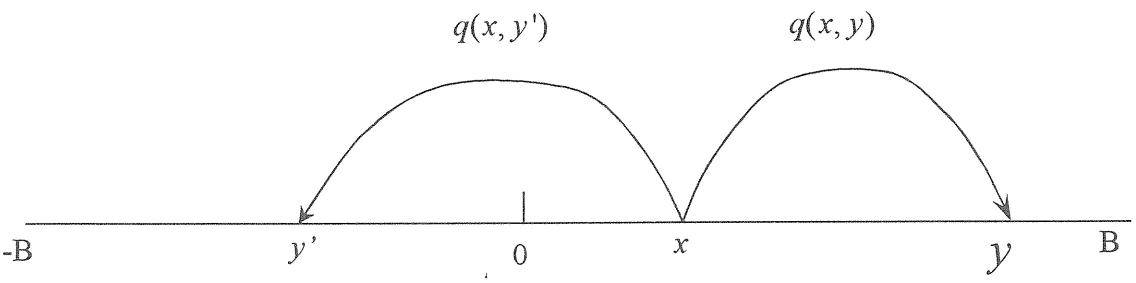


Figure 1　Setting of the model

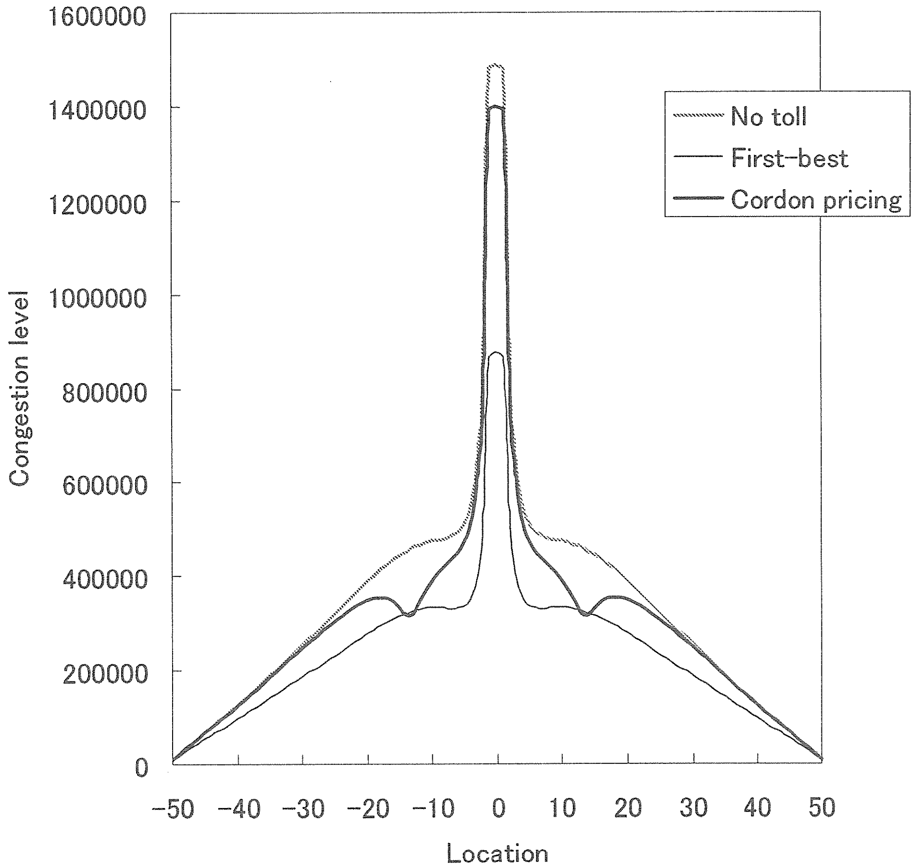
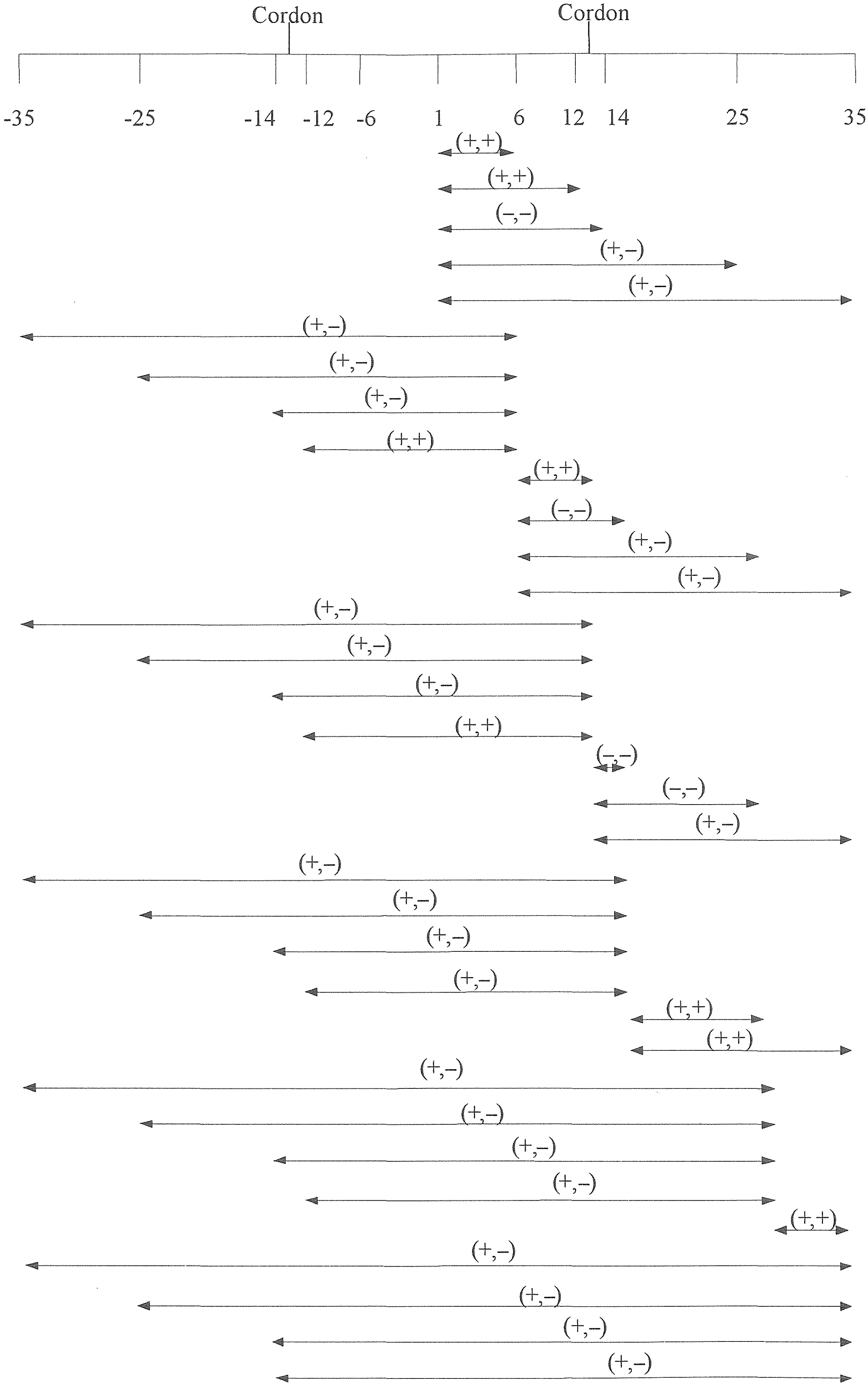


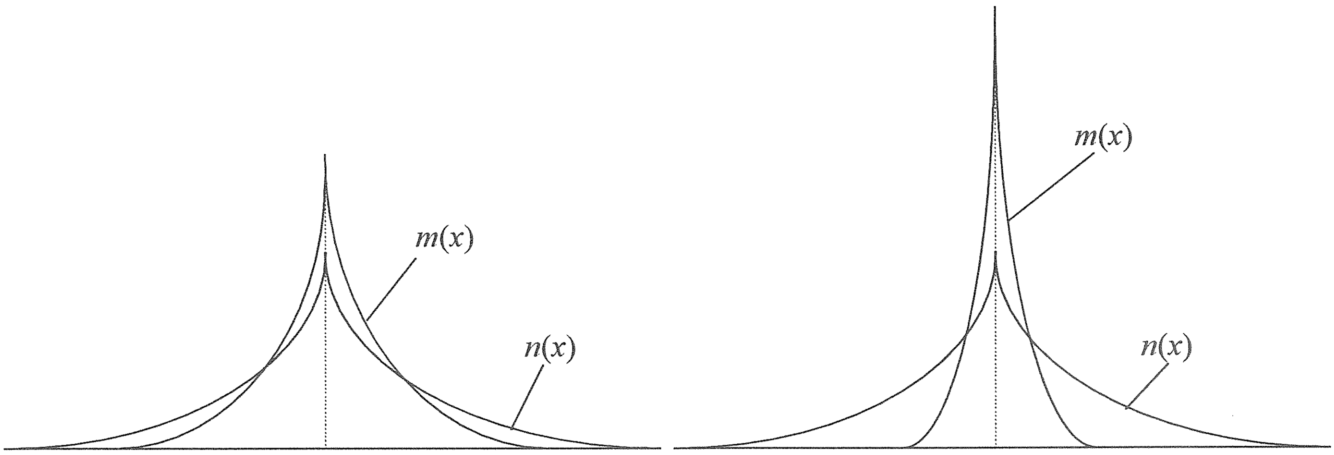
Figure 2 Spatial variation of congestion levels for three schemes



Note: signs indicated in the parentheses, (d_1, d_2), represents the deviations of trips as follows

$$d1 = Sgn\left(q^{**}(x,y) - q^{O}(x,y)\right) \quad d2 = Sgn\left(q^{**}(x,y) - q^{*}(x,y)\right)$$

Figure 3 Effects on the number of trips for selected O-D pairs



City A

City B

"City B is closer to monocentric than City A"

Figure 4　Difference in urban spatial structure

Table 1 Effects of urban spatial structure (1): gradient of $m(x)$

	Basic case	10g	50g
Cordon location	13	3	0
Toll	19	28	25
$SS_{No toll}$	7.731E+08	3.985E+08	1.621E+08
SS_{Cordon}	8.008E+08	4.603E+08	1.917E+08
$SS_{First best}$	8.397E+08	4.864E+08	1.939E+08
Maximal gain	0.086	0.220	0.196
Gain from cordon pricing	0.036	0.155	0.182
Relative gain	0.415	0.703	0.931

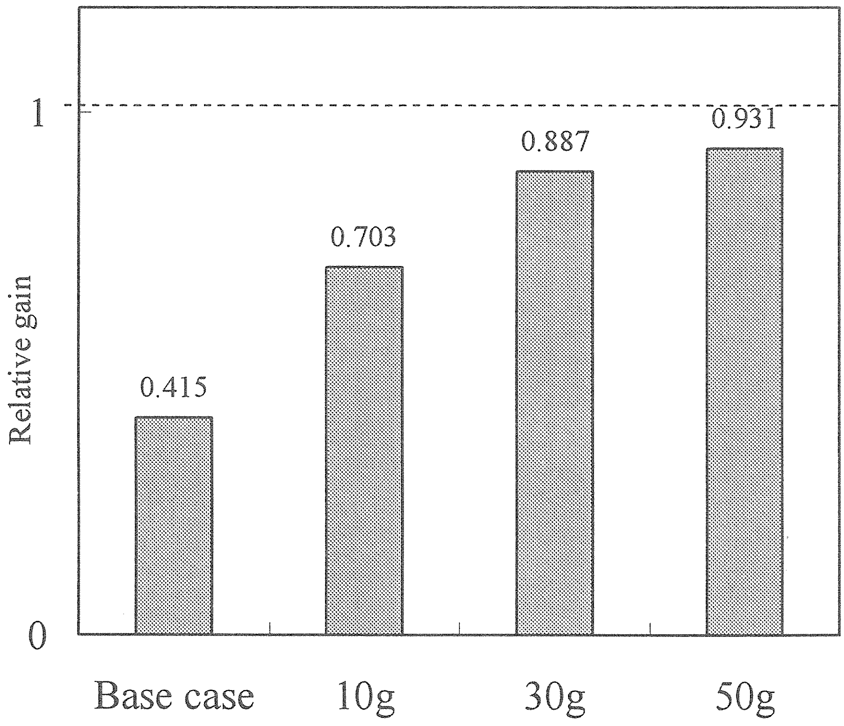


Figure 5 Monocentricity and relative gain of cordon pricing

Table 2 Effects of urban spatial structure (2): overall desnsity gradient

	$0.5g$	Basic case	$1.5g$
Cordon location	23	13	8
Toll	18	19	19
$SS_{No toll}$	7.505E+08	7.731E+08	8.119E+08
SS_{Cordon}	7.743E+08	8.008E+08	8.452E+08
$SS_{First best}$	8.093E+08	8.397E+08	8.878E+08
Maximal gain	0.078	0.086	0.093
Gain from cordon pricing	0.032	0.036	0.041
Relative gain	0.405	0.415	0.438

Table 3 Effects of demand elasticity

	$0.5a$	Basic case	$1.5a$
Cordon location	8	13	14
Toll	17	19	20
$SS_{No toll}$	4.643E+08	7.731E+08	1.032E+09
SS_{Cordon}	4.798E+08	8.008E+08	1.070E+09
$SS_{First best}$	4.964E+08	8.397E+08	1.131E+09
Maximal gain	0.069	0.086	0.096
Gain from cordon pricing	0.034	0.036	0.037
Relative gain	0.485	0.415	0.384

Table 4 Effects of road capacity

	$0.5k$	Basic case	$1.5k$
Cordon location	14	13	11
Toll	21	19	17
$SS_{No\,toll}$	6.313E+08	7.731E+08	8.627E+08
SS_{Cordon}	6.547E+08	8.008E+08	8.923E+08
$SS_{First\,best}$	6.964E+08	8.397E+08	9.284E+08
Maximal gain	0.103	0.086	0.076
Gain from cordon pricing	0.037	0.036	0.034
Relative gain	0.360	0.415	0.451

第４章　大阪都市圏における次善の料金政策に関する実証分析

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１．はじめに

前章までは、一次元の連続空間を対象として、コードンプライシングの効果を分析した。このような単純な空間設定は、問題のメカニズムを分析するために有益なものであったが、実際の都市の問題に対して具体的な知見を与えるには不十分である。特に実際の都市における交通は二次元の道路ネットワーク上を流れ、混雑の状況もネットワークの結節構造によって異なるものとなる。そして道路利用者は各リンクの混雑状況を考慮して経路を選択する。本章では、以上のような状況を記述するモデルを用いて、コードンプライシングの効果を分析する。

ネットワークを対象とした次善の料金政策に関する分析は、最近になっていくつか行われるようになった。May, Milne (2000) は、英国ケンブリッジ市のネットワークを対象としてコードン、距離比例制、時間比例制、システム最適を計算し、それらの比較を行った。コードン方式については、料金も料金徴収地点も与件であり、これは「次善」とはいえない。Santos, Newbery, Rojey (2001)は、英国の８都市でコードンプライシングの効果を計算した。そこでは料金について最適化しているが、コードンの位置は固定していた。Verhoef (2002)はより一般的な問題を想定し、ネットワーク中の課金すべきリンクと料金水準の最適な組み合わせを求める問題を定式化し、小規模の例題ネットワークを対象にアルゴリズムの検討を行った。Zhang and Yang (2004)はネットワークにおける最適なコードンプライシング・システムを設計する問題を、最も厳密に定式化しその解法を提示している。すなわちコードンラインを横切るリンクの集合を内生的に求められるようにしている。そしてこの手法を上海のネットワークに適用し、コードンプライシングの厚生効果を計算している。

既存の研究において、コードンプライシングの効果は都市ごとに結果は異なっている。異なった結果が得られる要因としては、土地利用（都市の空間構造)、需要弾力性、交通容量、ネットワークの形状などが考えられる。仮想的な空間を対象とした前章の分析では、

特に都市の空間構造（単一中心的か否か）が結果に大きな影響を及ぼすことが示された。本研究では大阪都市圏を対象としたケーススタディを通じて、当該地域における導入効果を評価するとともに、世界各地で行われつつある実証的知見の蓄積にも貢献することを目的としている。

２．モデル

都市空間はＩ個の離散的なゾーンに区分されており、それぞれのゾーン中心ノードから交通需要の発生、集中がなされるものと仮定する。都市内の道路ネットワークはゾーン中心を含むノードとそれらを結ぶリンクから構成されており、ゾーン間の交通はネットワークに沿って流れる。ネットワークにはL本のリンクがあり、リンクaを流れる交通量をx_aと表す。交通需要は弾力的であり、ゾーンrからsへの交通需要は、逆需要関数$D_{rs}^{-1}(f)$によって与えられる。ゾーンrs間には複数の経路が存在し、トリップの費用は経路上の走行時間費用と道路料金の和（一般化費用）に等しいと仮定する。すなわち

$$c_{rs}^j = wt_{rs}^j + \tau_{rs}^j \tag{1}$$

ここにt_{rs}^jとτ_{rs}^jは、それぞれ、rs間のj番目の経路を選んだ場合の所要時間と道路料金、wは時間価値である。上式の右辺第１項は走行時間費用、第２項は道路料金であるが、いずれも選んだ経路（どのリンクを使って行くか）によって異なる。t_{rs}^jは経路上の各リンクの通過所要時間の和であるが、各リンクの通過所要時間はそのリンクを通る交通量の増加関数$t_a(x_a)$であると仮定する（以下これをリンク走行時間関数と呼ぶ）。したがって

$$t_{rs}^j = \sum_a \delta_{ars}^j t_a(x_a) \tag{2}$$

$$\tau_{rs}^j = \sum_a \delta_{ars}^j \tau_a \tag{3}$$

$$x_a = \sum_r \sum_s \sum_j \delta_{ars}^j f_{rs}^j \tag{4}$$

ここにf_{rs}^jはODペアrs間のj番目経路を選択したトリップの数、δ_{ars}^jはリンクaがODペアrs間のj番目経路上にある場合1、それ以外は0の値をとる変数である。またτ_aはリンクaに関する通行料金である。R_{rs}^jはrs間のj番目経路上にあるリンクの集合である。τ_aは料金政策によって異なるが、すべてのリンクについて正の値を持つわけではなく、実際に

はむしろゼロの値が与えられるリンクの方が多い。

いまODトリップ数Q_{rs}が与えられたものとする。各ドライバーは費用が最小となる経路を選択する。交通ネットワーク均衡は、どのドライバーも選択した経路を変更するインセンティブを持たないような状態が実現したとき達成される。このとき各ODについて、使われている経路のどれを選んでもトリップ費用はすべて等しい値となり、使われない経路の費用はその等しくなった費用より大きい。このことは、A_{rs}をODペアrs間で利用可能な経路の集合と定義すれば、すべての$j \in A_{rs}$について次の条件が成り立つことを意味する。

$$f_{rs}^{j} > 0 \quad \Rightarrow \quad c_{rs}^{j} = C_{rs}^{*}$$

$$f_{rs}^{j} = 0 \quad \Rightarrow \quad c_{rs}^{j} \geq C_{rs}^{*}$$

上の条件は、下記のように形式的に書き表される。

$$c_{rs}^{j} - C_{rs}^{*} \geq 0 \tag{5a}$$

$$f_{rs}^{j}(c_{rs}^{j} - C_{rs}^{*}) = 0 \tag{5b}$$

$$f_{rs}^{j} \geq 0 \tag{6}$$

また定義により次が成り立つ。

$$Q_{rs} = \sum_{j \in A_{rs}} f_{rs}^{j} \tag{7}$$

上式より、ODペアrs間のトリップに要する費用は（どの経路を選ぼうが）C_{rs}^{*}に等しくなる。各個人は、トリップを行うことによる私的便益がトリップ費用を上回る限りトリップを行うが、均衡においては、次の関係が成り立つようにrs間の総ODトリップ数Q_{rs}が決まる。

$$D_{rs}^{-1}(Q_{rs}) = C_{rs}^{*} \tag{8}$$

要するに、弾力的交通需要のもとでの交通ネットワーク均衡は、(5)-(8)を同時に満たすリンク交通量、経路交通量、OD交通量、均衡交通費用の組である。なお経路交通量は一意に決まらないが、リンク交通量が一意に決まればそれは問題にならない。リンク交通量からゾーン間走行費用も一意に求まり、それによって交通流の状態と厚生を評価するための十分な情報が与えられたことになる。

本稿ではベンチマークとして無料金均衡、すなわちどのリンクにまったく料金が徴収されないケースを考える。これは(3)式において$\tau_a = 0,\ \textit{for all } a$、と書ける。

一方、社会的に効率的な道路利用は、次に示すような社会的余剰最大化問題を解くことに

よって求められる。

$$\underset{q_{rs}^j}{Max} \quad \sum_r \sum_s \int_0^{Q_{rs}} D_{rs}^{-1}(z)dz - w\sum_a t_a(x_a)x_a \tag{9}$$

subject to (6)(7)

最適化の１階条件より、次の関係が導かれる。

$$D_{rs}^{-1}(Q_{rs}) - w\sum_a \delta_{ars}^j \left\{t_a(x_a) + t_a{}'(x_a)x_a\right\} \le 0 \tag{10a}$$

$$\left[D_{rs}^{-1}(Q_{rs}) - w\sum_a \delta_{ars}^j \left\{t_a(x_a) + t_a{}'(x_a)x_a\right\}\right] f_{rs}^j = 0 \tag{10b}$$

この条件を利用者均衡条件(5)-(8)と対応させると、各リンクにおける料金が

$$\tau_a = wt_a{}'(x_a)x_a \tag{11}$$

のように設定されるとき、上の最適条件が分権的な利用者均衡のもとで実現することがわかる。すなわち社会的に効率的な道路利用を達成するためには、ネットワークのすべてのリンクにおいて、混雑の外部効果に等しい料金を課する必要がある。このような料金を実行することは困難なので、実際には一部のリンクでのみ料金を徴収するシステムを採用せざるを得ない。

３．次善の料金政策

道路管理者は、料金徴収に伴う技術的、社会的制約のもとで、社会的余剰を最大化するよう、料金水準を定めるものとする。ここで考える次善の料金体系とは、ネットワークの一部のリンクでのみ料金が徴収可能であるという状況のもとで、それらのリンクで料金水準を最適に設定することである。なお料金が徴収されるリンクの集合は与えられるものとする。

上の定義に従えば、次善の料金体系は形式的には次の問題を解くことにより求められる。

$$\underset{\boldsymbol{\tau}}{Max} \quad \sum_r \sum_s \int_0^{Q_{rs}(\boldsymbol{\tau})} D_{rs}^{-1}(z)dz - w\sum_{a\in A} t_a(x_a(\boldsymbol{\tau}))x_a(\boldsymbol{\tau}) \tag{12}$$

subject to $\tau_a \ge 0,\ for\ a \in H,$ (13a)

$\tau_a = 0,\ for\ a \notin H$ (13b)

ここに $\boldsymbol{\tau} = (\tau_1, \tau_2, ..., \tau_L)$ であり、H は課金するリンクの集合（ただし $H \subset A$）である。また $Q_{rs}(\boldsymbol{\tau})$、$x_a(\boldsymbol{\tau})$ は、$\boldsymbol{\tau}$ のもとで利用者均衡条件(5)-(8)を解くことにより得られる。H に含まれ

るリンク以外では(13b)のような制約が課せられているので、上の問題の解はシステム最適よりも劣る次善の解となるのである。もし$H=A$であれば、上の問題の解はシステム最適と一致する。

コードンプライシングは、次善の料金体系の一特殊ケースである。まず単一のコードンプライシングでは、都市の中心部を取り囲むコードンを横切るすべてのリンクが課金されるリンク集合Hの要素となり、それらのリンクではすべて同額$\bar{\tau}^C$の料金が課される。したがって解くべき問題は次のように表される。

$$\underset{\bar{\tau}^C}{Max} \quad \sum_r \sum_s \int_0^{Q_{rs}(\boldsymbol{\tau})} D_{rs}^{-1}(z)dz - w\sum_{a\in A} t_a(x_a(\boldsymbol{\tau}))x_a(\boldsymbol{\tau}) \tag{14}$$

subject to $\tau_a = \bar{\tau}^C,\ for\ a \in H,$ (15a)

$$\tau_a = 0,\ for\ a \notin H \tag{15b}$$

次に多重のコードンプライシングについて述べる。M本のコードンラインが設定され、都市の中心部に近いものから$1,2,\ldots,M$のように番号が付けられているものとする。課金されるリンクの集合Hは、M個の部分集合$h_m,\ m=1,2,\ldots M$に分けられ、それぞれはm番目のコードンラインを横切るリンクの集合に対応する。そしてh_mに含まれるすべてのリンクでは同額の料金$\bar{\tau}^{C_m}$が適用される。多重コードンプライシングの場合は、上の問題(14)における制御変数$\bar{\tau}^C$が$\bar{\tau}^{C_m}$へとM次元になり、制約条件が下記のように置き換わる。

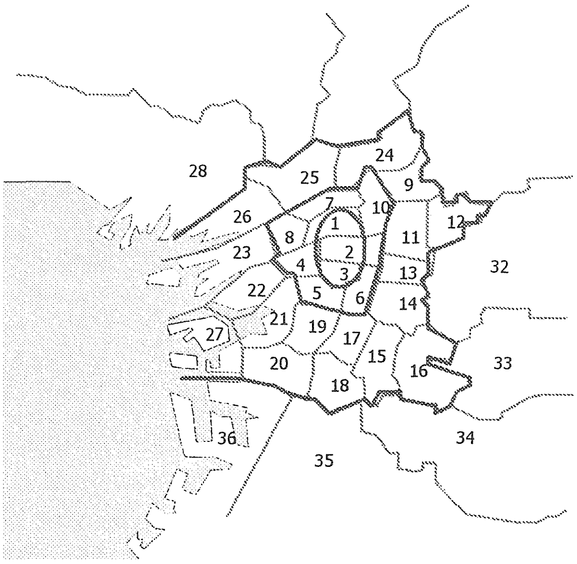
$$\tau_a = \bar{\tau}^{C_m}, \quad for\ a \in h_m, \quad \bigcup_{m=1}^{M} h_m = H \tag{16a}$$

$$\tau_a = 0, \quad for\ a \notin H \tag{16b}$$

４．大阪都市圏における実証分析

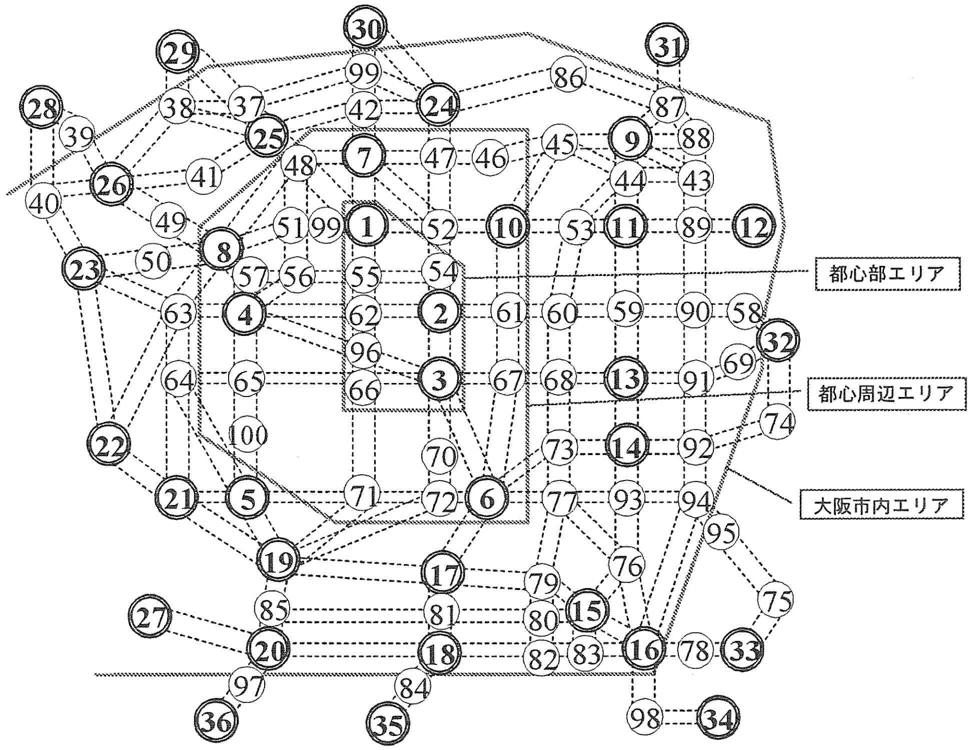
４－１　ゾーン区分とネットワーク

本研究では、大阪市内の２７区とその周辺の市町村を集約した９個のゾーンの合計３６ゾーンを対象とする。道路ネットワークは、阪神高速道路と一般道路を合わせて、241 ノード・630 リンクから成る。これらの詳細については、図－１と図－２に示している。またこれらの図には、後の分析で取り上げられるコードンの位置を太線で示している。

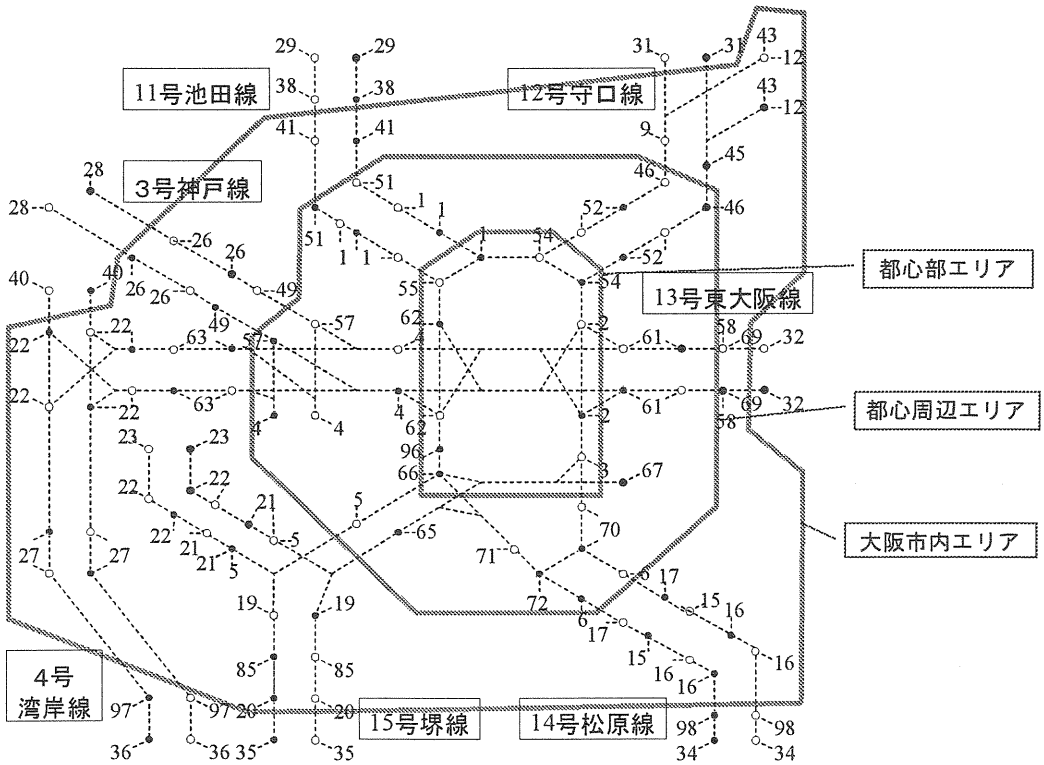


1	北区	19	西成区
2	中央区（旧東区）	20	住之江本区
3	中央区（旧南区）	21	大正区
4	西区	22	港区
5	浪速区	23	此花区
6	天王寺区	24	東淀川区
7	北区（旧大淀区）	25	淀川区
8	福島区	26	西淀川区
9	旭区	27	住之江南港
10	都島区	28	尼崎市以西
11	城東区	29	豊中市以西
12	鶴見区	30	吹田市以東
13	東成区	31	摂津・守口・門真市以東
14	生野区	32	東大阪・大東市以東
15	東住吉区	33	八尾市以東
16	平野区	34	松原市以東
17	阿倍野区	35	堺市内陸以南
18	住吉区	36	堺市臨海以南

図－1　対象地域とゾーン区分



(a) 一般道路ネットワーク



(b) 高速道路ネットワーク

図－２　計算に用いるネットワークとコードンの設定

4－2　リンク走行時間関数とトリップ需要関数

各リンクの走行時間関数を次のように特定化する。

$$t_a(x) = f_a\left\{1 + \nu\left(\frac{x_a}{K_a}\right)^{\gamma}\right\} \tag{17}$$

ここに f_a は自由走行状態でリンク a を通過するのに要する時間、K_a はリンク a の交通容量である。ここでは土木学会の交通需要予測小委員会が標準的な値として提案している、$\nu = 0.48,\ \gamma = 2.82$ を用いることとする。

需要関数は次式のように特定化する。

$$D_{rs}(C_{rs}) = \alpha \cdot n_r \cdot n_s^{\theta_S} \cdot \exp(-\beta \cdot C_{rs}) \tag{18}$$

ここに n_r, n_s は、それぞれ発ゾーン r および着ゾーン s における昼間人口である。α, β, θ_s はパラメータである。

上のような需要関数はしばしば交通量予測に適用される重力モデルと同じ形であるが、前章で示したように、個人の効用最大化行動から得られた需要関数を集計することにより導出することができる。逆需要関数は次式のようになる。

$$D_{rs}^{-1}(Q_{rs}) = -\frac{1}{\beta}\log\left(\frac{Q_{rs}}{\alpha \cdot n_r \cdot n_s^{\theta_S}}\right)$$

需要関数のパラメータ値は、観測値と適合するよう推定する。大阪都市圏における OD 交通量の観測値は 1994 年の全国道路交通センサスデータから得た。また時間費用と金銭的費用の和であるトリップ費用のデータは、以下のように作成した。

$$C_{rs} = \sum_{a \in A_{rs}^*}\{w t_a(\tilde{x}_a) + \tilde{\tau}_a\}$$

ここに $\tilde{x}_a, \tilde{\tau}_a$ は現状におけるリンク交通量の観測値と現行の道路料金（阪神高速道路）、A_{rs}^* は現状のリンク交通量と料金のもとで rs 間の（時間単位の）トリップ費用が最小となる経路に含まれるリンクの集合である。なお時間価値 w は 80 円／分・台を仮定する。これは阪神高速道路公団節約時間便益計算で用いられた値である。

需要関数のパラメータ推定結果は次の通りである。

$\alpha = 0.000024$　(15.0503)
$\theta_S = 0.6055$　(10.2505)　　　　$R^2 = 0.5430$
$\beta = -0.00074$ (−24.8606)

なお係数推定値の右側括弧内には t-値を示している。

4－3　無料金均衡とシステム最適

大阪都市圏における昼間人口とネットワークの条件、そして上のように求めたパラメータをモデルに与えて、無料金均衡とシステム最適について計算した結果を表－1に示す。

表－1　無料金均衡とシステム最適に関する計算結果

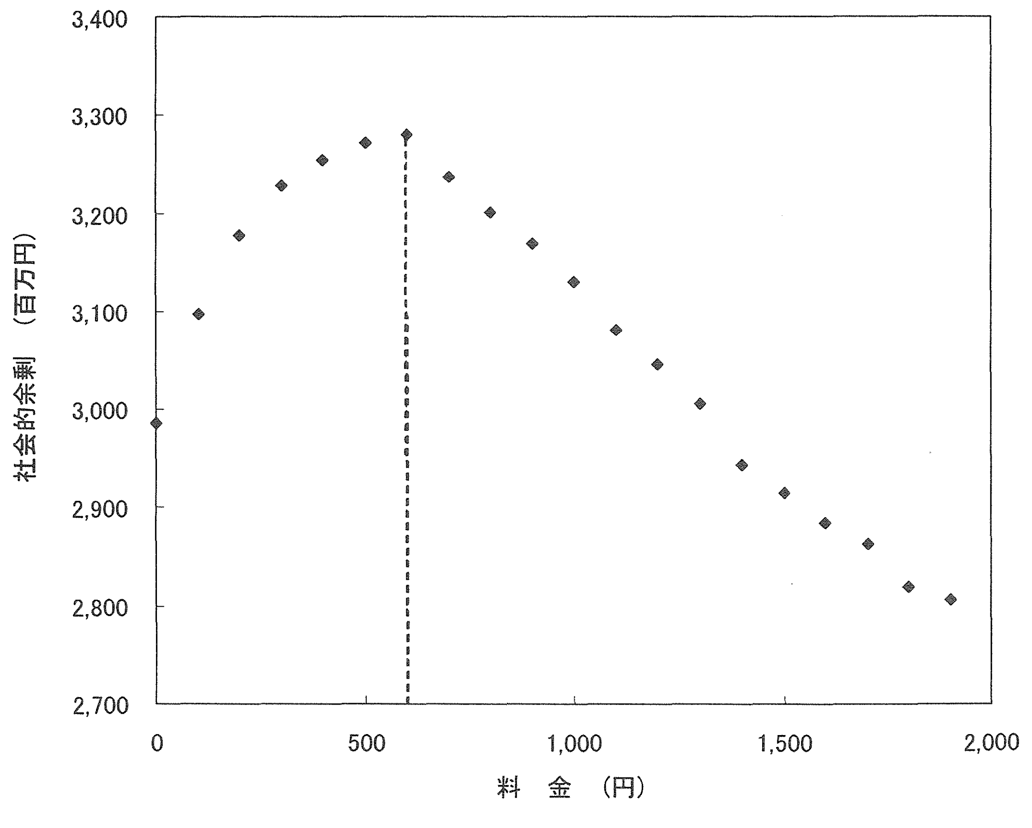
		無料金均衡	システム最適	変化率
総トリップ数	（台）	2,417,017	1,964,929	-18.70%
総走行距離	（台・キロ）	22,787,454	16,725,841	-26.60%
総走行時間	（台・時間）	1,217,712	690,078	-43.33%
平均トリップ長	（キロ）	9.43	8.51	-9.71%
平均旅行時間	（分）	30.2	21.1	-30.13%
平均料金支払額	（円）	0	901	
消費者余剰	（100万円）	2,985	2,143	-28.21%
混雑料金収入	（100万円）	0	1,771	
社会的余剰	（100万円）	2,985	3,914	31.12%

システム最適は、各リンクにおいて混雑外部効果に等しい料金を徴収することによって達成されるが、そのとき消費者余剰は無料金時に比べて 28%減少する。しかしその減少分を上回る料金収入を上げることができるため、社会的余剰は 31.12%増加する。そのような厚生改善は、総トリップ数を 18.7%削減することにより達成される。また平均トリップ長が減少することにも注目されたい。トリップ長が長いほど、その途上で他の道路利用者に混雑の外部効果を及ぼすが、それが内部化されない無料金均衡のもとではトリップ距離が過大となるのである。平均トリップ時間は 30.1%減少しているが、これと同じ率で速度上昇したことを意味しない。速度変化を求めるにはトリップ長の変化を考慮して補正する必要がある。すなわち無料金均衡時の平均速度は 9.43km/30.2 分=18.7km/時、システム最適時は 8.51km/21.1 分=24.2km/時なので、平均速度は 29.2%上昇したことになる。また「平均料金支払額」は 1 トリップあたりの料金額であるが、8.51km のトリップについて 901 円支払うことを意味する（1km あたり 106 円）。システム最適に比べると、無料金均衡における社会的余剰の値は 9 億 2900 万円少ないが、この値は交通混雑による一日あたりの損失額とみなすことができる。すなわち混雑外部性が内部化されて効率的な道路利用がなされていれば

達成されていた状況に比べ、無料金均衡ではその額だけ社会的に資源が浪費されていると考えることができる。またこの額は政策によって改善可能な便益の最大値といえるので、次善の政策によって、どの程度までこれに近づけることができるか、というのが評価の基準となる。

４－４　コードンプライシング

図－１、２に示すように、内側から都心部コードン、都心周辺部コードン、大阪市全域コードンという３通りのコードンを設定したが、まずはこれらの内一つのみで料金が徴収されるという、単一コードン料金制について検討する。前節の問題(14)(15)のように、社会的余剰を最大化するよう、それぞれの単一コードンにおいて徴収する料金額を求める。これは一次元の最適化である。また実際の適用を考慮すると料金額を厳密に１円単位で求めることは現実的ではないので、100 円単位で料金額を変化させて利用者均衡を計算し、それらの内社会的余剰の最大となる料金額を探索することにした。都心部コードンの場合、料金額と社会的余剰の関係は図－３のようになり、次善最適な料金は 600 円である。



図－３　都心部コードンにおける最適な料金

都心周辺部コードン、大阪市全域コードン料金制を実施した場合も同様にして最適な料金を求めた。それらの結果は表－3にまとめられている。都心周辺部コードンで料金を徴収する場合の最適料金は 800 円、大阪市全域の場合は 1100 円である。社会的余剰は大阪市全域を囲むコードンの場合が最大であり、無料金の場合に比べて 21.3%増加している。各コードンに関する結果を示す列の右側に示した数字は、無料金均衡に対する変化率である。ただし最下段の料金制導入便益は、コードン料金制導入した場合と無料金均衡の場合の社会的余剰との差であるが、その段の右側の数値は下の式で計算したものであり、最大限可能な厚生改善に対するコードン料金制の相対的効果を表している。

$$\frac{\text{（コードン料金のもとでの社会的余剰－無料金均衡のもとでの社会的余剰）}}{\text{（システム最適のもとでの社会的余剰－無料金均衡のもとでの社会的余剰）}}$$

最大の効果をもたらす大阪市全域コードンの場合、システム最適による改善の 69%の水準を達成できる。

次に多重コードンの場合について検討する。上記の 3 通りのコードンで料金を徴収する場合の料金の最適な組み合わせを求める。それぞれのコードンにおいて 0-2000 円の間で料金水準を 100 円単位で動かすと、組み合わせの数は$20^3 = 8000$通りである。この程度の数なら不可能な数ではないので、総当りで計算を実行した。社会的余剰の高いものから順位付けし、上位の組み合わせに関する結果を表－4 にまとめている。表によると、上位の組み合わせ間の社会的余剰の差は小さく、料金についてもかなり似通っている。最も内側の都心部コードンで 200-300 円、都心周辺コードンで 200-300 円、そして大阪市全域コードンで 800-1000 円となっている。大阪市外から都心部に入るため 3 つのコードンを通過すると、合計で 1300-1400 円の料金を支払わねばならない。表の最下段に示した、相対的改善の値を見ると、システム最適に対して、82%の厚生改善効果が達成できる。

表－３　三通りの単一コードンに関する結果の概要

		無料金均衡	都心コードン		都心周辺コードン		大阪市全域コードン	
最適料金	（円）		600		800		1100	
総トリップ数	（台）	2,417,017	2,199,024	-9.02%	2,079,387	-13.97%	2,080,653	-13.92%
総走行距離	（台・キロ）	22,787,454	19,987,245	-12.29%	18,316,267	-19.62%	17,414,877	-23.58%
総走行時間	（台・時間）	1,217,712	1,020,221	-16.22%	897,657	-26.28%	779,810	-35.96%
平均トリップ長	（キロ）	9.43	9.09	-3.59%	8.81	-6.57%	8.37	-11.22%
平均旅行時間	（分）	30.2	27.8	-7.91%	25.9	-14.31%	22.5	-25.61%
平均料金支払額	（円）		242		404		537	
消費者余剰	（100万円）	2,985	2,749	-7.91%	2,575	-13.75%	2,514	-15.80%
混雑料金収入	（100万円）	0	531		841		1,117	
社会的余剰	（100万円）	2,985	3,280	9.88%	3,416	14.42%	3,631	21.64%
料金制導入便益	（100万円）	0	295	31.76%	431	46.35%	646	69.52%

表－４　３重コードンのもとで上位の料金案に関する結果

		1	2	3	4	5
都心部コードン料金	（円）	200	200	200	300	300
都心周辺コードン料金	（円）	300	200	300	200	200
大阪市全域コードン料金	（円）	900	900	800	900	800
総トリップ数	（台）	1,909,297	1,955,271	1,940,314	1,917,478	1,949,301
総走行距離	（台・キロ）	15,526,012	16,068,349	15,972,431	15,628,140	16,077,300
総走行時間	（台・時間）	669,913	701,517	699,647	677,936	707,398
平均トリップ長	（キロ）	8.13	8.22	8.23	8.15	8.25
平均旅行時間	（分）	21.1	21.5	21.6	21.2	21.8
平均料金支払額	（円）	754	704	716	742	705
消費者余剰	（100万円）	2,309	2,368	2,354	2,319	2,367
混雑料金収入	（100万円）	1,440	1,377	1,390	1,423	1,374
社会的余剰	（100万円）	3,750	3,745	3,743	3,742	3,741
料金制導入便益	（100万円）	764	760	758	756	755
相対的改善		82.28%	81.80%	81.60%	81.42%	81.30%

単一コードンで最も大きな改善効果のある、大阪市全域コードンの結果を、これまで行われてきた、類似の研究事例と比較しよう。まず本研究で得られた結果を要約すると次の通りである。

総トリップ数 13.9%減少

社会的余剰は 646 百万円(21.6%)増加

システム最適に対する相対的改善は 69.5%

Santos 他(2000)は、英国の 8 都市を対象として、本研究とほぼ同様の手法によりコードン料金の効果を分析した。結果は都市によって大きな差があり、最適な料金水準は 0.25-3.5 ポンドと求められた。そしてこのような料金を課した結果、トリップ数は 0.8-8.4%減少、1pcu・km あたり社会的余剰が 0.7-7.3 ペンスであるという結果を示している。社会的余剰に関する結果を比較するため、本研究における社会的余剰の増加分を総走行距離で割り、さらに時間価値[1]で割ることにより、時間単位で km あたり余剰の増加分を求めた。英国の数値は 0.03-0.31 分であるのに比べ、本研究は 0.46 分であり、英国の 8 都市における最大値よりもさらに大きい。トリップ数の減少率についても同様に、本研究における結果がかなり大きい。

Zhang and Yang (2004)は上海のネットワークを対象として、単一コードンおよび二重コードン料金制に関する分析を行った。彼らはネットワークにおけるコードンラインの位置に関する最適化も行っている。ケーススタディでは朝のラッシュ時を想定してすべてのトリップが都心ノードに吸収されるものとしており、またコードンラインの外側から流入する場合のみ課金を行っている。計算の結果、単一コードンについて最適化した場合に社会的余剰が 0.93%の増加にとどまり、システム最適に対する相対的改善率も 28.3%であった。これは単一中心性を仮定したことを考慮すれば、小さい値といえる。本研究では非単一中心を想定しているにもかかわらず、Zhang and Yang (2004)の結果と比べて、厚生の改善効果がかなり大きい。

Sumalee (2004)は英国のエジンバラを対象として、単一コードンの位置と料金の最適な組み合わせを求めている。ここではシステム最適に対する相対的改善率はさらに小さく、20.5%であった。

前章で示したように、料金政策の効果は都市構造、需要の弾力性、ネットワークの整備

[1] Santos らは 23.4 ペンス／pcu・分、本研究では 80 円／台・分を仮定している。

水準などに大きく依存する。実際、Santos らの研究では、同じ分析手法を用いても料金政策の効果が都市ごとに大きく異なっていることから、本研究における効果が異常に大きいものなのか判断することは困難である。考えられる要因として次の点が指摘できる。

(a)本研究では、コードンを横切る方向にかかわらず一定の額の料金を徴収することにしている。他の研究では、都市の外側から内側に流入する方向のみに料金を課している。

(b)本研究において推定された需要関数は、トリップ費用に関する弾力性が大きい。(18)式によれば需要の価格弾力性はβC_{rs}により求められる。ここにパラメータ推定値と平均トリップ費用（平均トリップ時間 30.2 分×時間価値 80 円/分）を適用すると、弾力性値は 1.78 となる。この値は他の研究で用いられている値に比べてかなり大きい。

5．おわりに

本研究では、ネットワークにおける混雑と交通流動を記述するモデルを大阪都市圏に適用し、コードンプライシングの効果を実証的に分析した。都心部、都心周辺、大阪市全域を囲む３通りのコードン案について最適な料金を求め、社会的余剰の改善効果を評価した。その結果、大阪市全域コードンが最も高い厚生改善効果をもたらすことが示された。また３コードンで同時に料金を徴収する 3 重コードン制のもとで最適な料金の組み合わせを求めたところ、大阪市外から都心に行くためにすべてのコードンを通過する場合、合計 1300-1400 円の料金を徴収することが最適であることが示された。またそのとき、システム最適による厚生改善の 80%もの効果を達成することが示された。

本研究ではコードンプライシングのみを検討の対象としたが、他にも代替的な料金政策をいくつか考えることができる。特に日本の大都市（東京、大阪）では、都市内に高速道路ネットワークが存在し、そこですでに料金徴収が行われていることに注目してもよい。ETC などを前提とすれば、現行の均一料金制を改め、より柔軟な高速道路料金体系を採用することによって、有効な混雑対策になりうるかもしれない。少なくとも社会的受容性を考慮すれば、コードンプライシングよりは導入が容易であると思われる。これらの問題に関するさらなる研究は今後の課題としたい。

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